Log-linear approximation versus an exact solution at the ZLB in the New Keynesian model*

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Abstract

How accurate is a log-linear approximation of the New Keynesian model when the nominal interest rate is bounded by zero? This paper compares the solution of the exact non-linear model to the log-linear approximation. It finds that the difference is modest for a common economic scenario. This applies even for extreme events in numerical experiments that replicate the U.S. Great Depression. The exact non-linear model makes the same predictions as the log-linear approximation for key policy questions such as the size and sign of government spending and tax multipliers. It also replicates well known paradoxes like the paradox of toil and the paradox of price flexibility. The paper also reconciles different findings reported in the literature using Calvo versus Rotemberg pricing.

Keywords: non-linearities, zero lower bound, liquidity trap, Calvo price setting.

JEL Classification: E30, E50, E60

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1 Introduction

Short-term interest rate collapsed to zero in most of the advanced world following the crisis of 2008, which limited central banks' ability to stimulate spending. This shifted the focus of policymakers and researchers to fiscal policy. Some striking results emerge from a class of New Keynesian models, typically using the Calvo (1983) model of price frictions, see e.g. Eggertsson (2011), Christiano, Eichenbaum and Rebelo (2011), and Woodford (2011).[1] Once the zero bound on the short term nominal interest rate (ZLB) is binding, and if the monetary authority cannot commit to future monetary policy, the multiplier of government spending is always larger than one and in some cases much larger than one. Another noteworthy result, see e.g. Eggertsson (2010), is that supply side policies such as labor tax cuts are contractionary at the ZLB (also known as the paradox of toil). While the former result appeared to resurrect old Keynesian orthodoxy, the latter one goes against much of the conventional wisdom in macroeconomics.

There has been growing skepticism about the accuracy of these results in the economic profession. The work cited above relies on log-linear approximations of all equilibrium conditions except it explicitly keeps track of the constraint that the short term nominal interest rate is bounded by zero.[2] Boneva, Braun and Waki (2016) report results from a non-linear calibration of a model with quadratic pricing frictions (based on Rotemberg 1982). They find that government spending multipliers are small and orthodox at the ZLB, while the paradox of toil disappears. These results are of great interest, since the Calvo and the Rotemberg model are equivalent in their log-linear approximation. Fernández-Villaverde, Gordon, Guerrón-Quintana and Rubio-Ramirez (2015) study the non-linear Calvo model and emphasize the importance of non-linear features brought about due to the ZLB. A key contribution of their work is to consider richer structure for uncertainty than the previous literature, which clarifies several important non-linear properties implied by the ZLB that the previous literature did not capture. Miao and Ngo (2016) compare the fully non-linear Calvo model with the Rotemberg pricing model under uncertainty and document that these two models behave very differently in a non-linear setting. Taken together, these papers raise doubts about the accuracy of the existing literature that relies on log-linearizations, about the effect of fiscal policy at the ZLB.

One difficulty in making an overall assessment about the accuracy of the log-linearization in papers such as Eggertsson (2011), Christiano, Eichenbaum and Rebelo (2011), and Woodford (2011) on the basis of the work cited above, is that it either uses a different form of uncertainty, a different price mechanism (Calvo vs Rotemberg) or some combination of the two. This paper

[1] The first paper to formally study fiscal policy in the New Keynesian model subject to the ZLB is Eggertsson (2001).

[2] It is not important if the bound is exactly zero, or slightly positive or negative. The key is that there is *a bound*.

fills this gap.

The main finding of this paper is that the difference between solving the fully non-linear model or using a log-linear approximation (but taking account of the non-linearity created by the ZLB) is small in the numerical experiments but with important caveats. The exact non-linear Calvo model is solved and compared to a log-linear approximation, using the same form of uncertainty as in Eggertsson (2011), Christiano, Eichenbaum and Rebelo (2011), and Woodford (2011), i.e. a two state Markov process with an absorbing state. Moreover, exactly the same procedure is used to parameterize the size of the shocks and parameters in the log-linear and the non-linear solutions of the model.

Table 1: Multipliers at the ZLB

Calibrations	GD in Non-Linear	GD in Linear	GR in Non Linear	GR in Linear
$\frac{\partial \hat{Y}}{\partial G}$	2.4166	2.2168	1.2905	1.1828
$\frac{\partial \hat{Y}}{\partial \tau^w}$	1.5889	0.9953	0.1198	0.1499

The bottomline is summarized in Table 1 that considers two benchmark parameterizations. One numerical experiment matches a recession of the order of the Great Recession (GR) (an output gap of -10 percent and a two percent fall in inflation). The other experiment matches a large contraction of the order of the Great Depression (GD) (30 percent output gap and a 10 percent drop in inflation). As the table reveals, there is little difference between the log-linear approximation of the model and the exact non-linear solution: the government spending multiplier is above one in the GR scenario and above 2 in the GD scenario as in Eggertsson (2011). In addition, the paradox of toil does not depend on the log-linear approximation, it occurs in either setting.

Overall, the results suggest that the log-linear approximation is not driving the often cited results of fiscal policy in New Keynesian model at the ZLB. The key non-linearity to keep track of is the ZLB, while the remainder of the model appears to be well approximated with log-linear approximation, at least under the assumption of a two state Markov process with an absorbing state. This does not, however, imply that there are no important non-linearities created by richer stochastic structure for the shocks at the ZLB, as documented by Erceg and Lindé (2014) and Fernández-Villaverde et al. (2015).

The assumption that uncertainty is captured by a two state Markov process with an absorbing state implies that the model can be collapsed into only two equations (aggregate demand and aggregate supply) in two unknowns (inflation and output). This makes the computation of the equilibrium in the non-linear model simple. An important assumption that allows this characterization is industry-specific labor markets as in Eggertsson and Woodford (2003). This assumption eliminates price dispersion as a state variable.[3] Fernández-Villaverde et al. (2015) and Miao and Ngo (2016), however, assume a common labor market from which all firms must hire, in which case price dispersion is a state variable of the non-linear model. Consequently a simple two equation characterization of the form presented in this paper is not possible (see Woodford (2003) for a general discussion of the importance of this assumption in New Keynesian models).

Another important assumption maintained in this paper is that the fiscal policy intervention is perfectly correlated with the shock that drives the economy to the ZLB, i.e. the fiscal stimulus is in response to the recession. As emphasized in Eggertsson (2011), fiscal policy has a very different effect if it follows a different stochastic process than the shock that drives the economy to the ZLB. To take an extreme example, Eggertsson (2011) characterizes analytically when the government increases spending permanently at the ZLB. In this case the fiscal multiplier changes signs and becomes negative. Being explicit about the correlation of government spending with the underlying ZLB shock process is an important consideration to reconcile results reported in the literature that may sometime seem to conflict each other.

Finally, the main focus in this paper is the analysis of ZLB equilibrium that arises due to real shocks that lead to the ZLB being binding. The paper does not analyze equilibria that arise due to self-fulfilling expectations, but these type of equilibria can arise whether or not the model is log-linearized. There is a rich literature exploring this topic, an important recent contribution includes Christiano, Eichenbaum and Johannsen (2018).[4] Christiano, Eichenbaum and Johannsen (2018)'s results suggest that the equilibria that arise due to self-fulfilling expectations can be ruled out via stability-under-learning criteria.[5]

While the log-linearization approximates the non-linear model at the ZLB quite well, when an equilibrium exists, the non-linear variation of the model also offers some additional insights that are briefly commented on below where the organization of the paper is outlined, with more details to follow.

Section 2 states the non-linear Calvo model. Section 2.1 clarifies the role of industry specific labor markets. While the framework is standard, it is useful to clarify why price dispersion does

[3]This modeling assumption is relatively standard in the New Keynesian literature, see e.g. Woodford (2003) for the textbook treatment. However, as stressed by Benigno and Woodford (2004), price dispersion is needed for welfare analysis and hence will feature as a state variable in the analysis of optimal monetary policy.

[4]Aruoba, Cuba-Borda and Schorfheide (2017), Benhabib, Schmitt-Grohé and Uribe (2001), Christiano and Eichenbaum (2012), Mertens and Ravn (2014), and Schmitt-Grohé and Uribe (2017), are some examples of important contributions in this literature. We do show, however, that the non-linear model exhibits multiple equilibria of a different form when the zero lower bound is binding, but we restrict attention to the locally determinate equilibrium, which allows us to do comparative statics and address questions related to multipliers.

[5]While their main focus is on ruling out multiplicity they also report numerical results on government spending multipliers in a Calvo model with common factor market for labor. Their numerical experiments deliver similar results across the linear and non-linear model. This suggests that the assumption of industry specific labor market, while helpful for tractability, is not driving the results reported in this paper.

not appear in the definition of equilibrium. Section 2.2 summarizes the non-linear model with two equations in two unknown, i.e. output and inflation, given fiscal policy and an exogenous shock that follows a two state Markov process. This section shows that the non-linear model typically has two solutions, the "regular ZLB" which is similar to the one analyzed in the log-linearized models, and a hyper-deflation equilibrium. As already noted, the paper does not consider the case in which the ZLB is binding due to self-fulfilling expectations, which represents a third possibility. The hyper-deflation equilibrium appears less interesting than the "regular ZLB" as it is shown not to pass standard equilibria selection criteria like local-determinacy and e-learnability, in which case comparative statics are less meaningful.[6] Accordingly the paper focuses on the "regular ZLB" equilibria and leaves the study of indeterminate hyper-deflation equilibrium to future work.

Section 3 documents, in line with the literature that relies on log-linearization, that the drop in output is increasing in the persistence of the shock. However, while at some critical value the equilibrium in the log-linearized model "explodes" and then becomes indeterminate, the analog in the non-linear model is a finite drop in output which is followed by a non-existence of equilibria. This gives a new rationale for the relative slope condition of the aggregate demand and aggregate supply, discussed in Eggertsson (2011). In the non-linear solution, the model tends to converge to a finite limit as the prices are made increasingly flexible or the ZLB shock is made persistent.

Section 4 compares the quantitative predictions of the log-linear approximation and the exact solution, i.e., what underlies Table 1 and explains the way in which the model is parameterized. Finally, section 5 clarifies why the non-linear Calvo model behaves so differently from the non-linear Rotemberg model documented in Boneva, Braun and Waki (2016) and Miao and Ngo (2016).[7] The key reason is that in the Rotemberg model, large deflation implies very large resource cost of price adjustment. The section documents that the policy disagreement between the Calvo and the Rotemberg model is an artifact of a particular way of accounting for the adjustment costs The resource costs of price changes become implausibly large very quickly (over 100% of GDP for 10% deflation) suggesting that the model with Rotemberg price adjustment costs cannot generate the Great Depression. However, there exist suitable modifications to this model, often applied in the literature, that can overcome this counterfactural policy disagreement. Motivated

[6]When an equilibrium is indeterminate in the model it means that there is an infinite number of bounded equilibria that solve the equations of the model in the vicinity of that equilibrium. This means that comparative statics are not meaningful absent further assumptions, for it becomes arbitrary as to which equilibria to select in response to an exogenous disturbance.

[7]Some of the equilibria analyzed by Boneva, Braun and Waki (2016) in the Rotemberg model are not e-learnable. We do not consider those equilibria. Our focus is only on the policy disagreements generated by the locally unique equilibria in the Rotemberg model since the comparative statics are defined here without the need for additional assumptions.

by Rotemberg's original article, this section concludes by suggesting an alternative interpretation of these costs so that these costs do not show up directly in the resource constraint.

2 Non-linear Calvo Model at the ZLB

2.1 Basic Model

This section briefly outlines the textbook (Woodford, 2003) New Keynesian model highlighting the relevance of industry specific market for wages. It concludes with a proposition summarizing that the model does not have any endogenous state variable, provided that monetary policy is purely forward looking. The representative household maximizes

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left[\frac{C_t^{1-\tilde{\sigma}^{-1}}}{1-\tilde{\sigma}^{-1}} - \lambda \frac{\int_0^1 n_t(i)^{1+\omega} di}{1+\omega} \right]$$

where C_t is a Dixit-Stiglitz aggregate

$$C_t = \left[\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$$

with $\theta > 1$, $n_t(i)$ is the quantity of labor supplied of *type* i, $\omega > 0$ is the inverse of the Frisch elasticity of labor, λ a normalizing constant and $0 < \beta < 1$ the discount factor.

The household demand for good i is

$$c_t(i) = C_t \left(\frac{p_t(i)}{P_t} \right)^{-\theta}$$

where $p_t(i)$ is the price of good i, P_t is the price level given by

$$P_t \equiv \left[\int_0^1 p_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}$$

so that $P_t C_t = \int_0^1 p_t(i) c_t(i) di$.

The households trade in one-period risk-free nominal bonds B_t that pay $1 + i_t$ in the next period where i_t is the nominal interest rate.[8] The period flow budget constraint is:

$$\int_0^1 p_t(i) c_t(i) di + B_t \leq (1 + i_{t-1}) B_{t-1} + (1 - \tau_t^w) \int_0^1 w_t(i) n_t(i) di + \int_0^1 Z_t(i) di - T_t,$$

[8] It is not important here whether the markets are assumed to be complete or if the only asset is a one period risk-free nominal bond, whose existence is needed to price the nominal interest rate which is the instrument of monetary policy.

where $w_t(i)$ is the nominal wage rate in the ith industry in the economy, $Z_t(i)$ is the nominal profits from the sale of good i, T_t are the lump sum taxes and τ_t^w is the payroll tax.

Every good i is produced by a monopolistically competitive firm. The production function of the firm is assumed to be linear in industry-specific-labor:

$$y_t(i) = n_t(i)$$

All production is consumed, so that $C_t = Y_t$, and each firm faces the demand function for its production given by $y_t(i) = Y_t(\frac{p_t(i)}{P_t})^{-\theta}$.

The key assumption in Woodford (2003) is that the labor employed by each monopolistically competitive firm corresponds to a particular type of the labor variety supplied by the households. The firm takes this wage rate *as given*. The firm's period profit function is then given by

$$\Pi_t(i) = p_t(i) \left(\frac{p_t(i)}{P_t}\right)^{-\theta} Y_t - w_t^I \left(\frac{p_t(i)}{P_t}\right)^{-\theta} Y_t.$$

Woodford (2003) suggests that w_t^I should be interpreted as an industry specific wage for good variety i. This wage rate can then be related to the price level of good i in that industry via the first order labor condition of the household as $\frac{w_t^I}{P_t} = \frac{\lambda n_t(i)^\omega C_t^{\tilde{\sigma}-1}}{(1-\tau^w)} = \frac{\lambda Y_t^\omega (\frac{p_t^I}{P_t})^{-\theta\omega} C_t^{\tilde{\sigma}-1}}{(1-\tau^w)}$ where p_t^I is the industry-wide common price. Accordingly the period profit function of a firm producing good variety i can be written as $\Pi\left(p_t(i), p_t^I, P_t, Y_t\right)$.[9]

Using the Calvo (1983) price-setting assumption, a fraction $0 < \alpha < 1$ of good prices remain unchanged in any period. With probability $1-\alpha$, a firm is selected at random (independent of the time of last adjustment) to adjust its good's price. Let p_t^* be the optimal reset price in period t. A supplier that changes its price in period t chooses its new price $p_t(i)$ to maximize present discounted value of its profits, taking as given the industry level wage w_t^I which is expressed in terms of p_t^I:

$$\mathbb{E}_t \left\{ \sum_{j=t}^{\infty} \alpha^{j-t} Q_{t,j} \tilde{\Pi} \left(p_t(i), p_j^I, P_j, Y_j \right) \right\}$$

where $Q_{t,t+1}$ is the stochastic discount factor in period $t+1$ with respect to t. The first order

[9] As noted by Woodford (2003), even if industry-specific labor markets are assumed, this need not imply that firms have monopsony power. Instead it can be assumed that each firm is a wage-taker as noted above. See page 148 in chapter 3 in Woodford (2003) for an elaboration on this point but where a more general notation is also developed to make this point clear: In short, assume a double continuum of differentiated goods, indexed by (I, j), with an elasticity of substitution of θ between any two goods. Then all goods with the same index I (goods in the same "industry") are assumed to change their prices at the same time, all using the same type I labor. Gertler and Leahy (2008) use a similar structure to this double continuum, except they allow for idiosyncratic risk across firms within an 'island' (industry).

condition for optimal price setting is:

$$\mathbb{E}_t \left\{ \sum_{j=t}^{\infty} \alpha^{j-t} Q_{t,j} \tilde{\Pi}_1 \left(p_t^*, p_j^I, P_j, Y_j \right) \right\} = 0$$

Following (Woodford, 2003) all firms in industry I reset the price in period t so this becomes:

$$\mathbb{E}_t \left\{ \sum_{j=t}^{\infty} \alpha^{j-t} Q_{t,j} \tilde{\Pi}_1 \left(p_t^*, p_t^*, P_j, Y_j \right) \right\} = 0$$

This gives the closed form solution :

$$\frac{p_t^*}{P_t} = \left(\frac{K_t}{F_t} \right)^{\frac{1}{1+\omega\theta}}$$

where F_t and K_t are aggregate variables given by:

$$F_t \equiv \mathbb{E}_t \sum_{j=t}^{\infty} (\alpha\beta)^{j-t} \xi_j (1 - \tau_j^w) C_j^{-\tilde{\sigma}^{-1}} Y_j \left(\frac{P_j}{P_t} \right)^{\theta-1}$$

$$K_t \equiv \mathbb{E}_t \sum_{j=t}^{\infty} (\alpha\beta)^{j-t} \frac{\theta}{\theta-1} \lambda \xi_j Y_j^{1+\omega} \left(\frac{P_j}{P_t} \right)^{\theta(1+\omega)}$$

To summarize, the following equations define the equilibria

1. Euler Equation

$$1 = \beta(1+i_t)\mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-1/\tilde{\sigma}} \frac{\xi_{t+1}}{\xi_t} \frac{1}{\Pi_{t+1}} \right] \tag{1}$$

2. NK Phillips Curve

$$K_t = \frac{\theta}{\theta-1} \lambda \xi_t Y_t^{1+\omega} + \alpha\beta\mathbb{E}_t \left[\Pi_{t+1}^{\theta(1+\omega)} K_{t+1} \right] \tag{2}$$

$$F_t = \xi_t (1 - \tau_t^w) C_t^{-\frac{1}{\tilde{\sigma}}} Y_t + \alpha\beta\mathbb{E}_t \left[\Pi_{t+1}^{(\theta-1)} F_{t+1} \right] \tag{3}$$

$$\frac{K_t}{F_t} = \left(\frac{1 - \alpha\Pi_t^{\theta-1}}{1-\alpha} \right)^{\frac{1+\omega\theta}{1-\theta}} \tag{4}$$

3. Resource Constraint

$$Y_t = C_t + G_t \tag{5}$$

Finally, price dispersion at time t is defined as

$$\Delta_t \equiv \int_0^1 \left(\frac{p_t(i)}{P_t}\right)^{-\theta(1+\omega)} di \tag{6}$$

To summarize:

Proposition 1. *In a representative agent New Keynesian economy with industry-specific labor markets and Calvo price setting, an equilibrium is defined as a collection of stochastic processes for* $\{C_t, Y_t, \Pi_t, K_t, F_t\}$ *that solve eqns* $\{1 - 5\}$ *given a path for* $\{G_t, \tau_t^w\}$ *determined by fiscal policy, a path for* $\{i_t\}$ *determined by monetary policy and a path for* $\{\xi_t\}$*. Price dispersion defined by eqn 6 does not feature as a state variable and is redundant in the determination of the equilibrium as long as it does not enter the government's policy (central bank and treasury's policy) choices.*

The reason price dispersion does not enter the determination of the endogenous variables is pointed out in Woodford (2003). The benchmark New Keynesian model assumes industry-specific labor markets so that the real marginal cost of supplying a given good depends only on $y_t(i)$ and aggregate output Y_t. Alternatively in a model in which each firm hires from an economy-wide labor markets, then the real marginal cost instead depends also on an alternative measure of aggregate output $X_t = \int_0^1 y_t(i)di$ which in turn depends upon price dispersion (Yun, 1996).[10] Using industry-specific labor markets instead of economy-wide factor markets has important implications for real rigidities in the economy as stressed in Woodford (2003). It also allows for the analysis of the model without price dispersion as a state variable, as long as the government policy does not depend on price dispersion. For welfare evaluation, however, price dispersion is relevant, for it enters social welfare. Accordingly, optimal monetary policy will in general take price dispersion into account (see Benigno and Woodford 2004). In this paper, however, policy will be characterized by a strict inflation target without reference to any endogenous state variable, as in Eggertsson and Woodford (2003).

2.2 Two state Markov Chain, the ZLB and AS and AD equations

Closed form solutions for 'Aggregate Supply' and 'Aggregate Demand' can now be derived under the assumption that the exogenous variable can only take on two values. ξ_t is a two-state Markov chain with an absorbing state. The long run , as in Eggertsson (2011), is when the shocks are at the non-stochastic steady state. The long run is an absorbing state. The short run, in

[10]Compare the intra-temporal condition of the household in the model with industry-specific labor markets with the economy-wide labor markets. $\frac{W_t}{P_t}$ depends on N_t in the economy wide labor markets model, and this aggregate depends on the price dispersion. On the other hand, the factor-market-specificity assumption makes the real wage independent of this aggregate.

contrast, is defined by period $t \geq 0$ at which time the shock ξ_S takes on a value below its steady state value $\xi_{L.}$ It is assumed that in each period $t < T^e$ when $\xi_t = \xi_S$ there is a constant probability $(1-\mu)$ that the shock reverts to its long run steady state value. $T^e < \infty$ is the stochastic (but finite) time period when the shock is back at the steady state level. So $t \geq T^e$ is defined as the long run.

The central bank sets an inflation target $\Pi_t = 1$ whenever it can. If this inflation target implies negative nominal interest rate, then the central bank sets $i_t = 0$ in which case Π_t is endogenously determined.[11]The analysis only considers the case in which the ZLB is binding due to fundamental shocks but does not consider the possibility that the ZLB is binding due to self-fulfilling expectations. Government spending and/or labor taxes can take on any values G_S and τ_S^w in the short run but return back to steady state in the long run given by G_L and τ_L^w. There is perfect correlation of the fiscal policy intervention and ξ_t.[12] Long run government spending G_L is calibrated at 0.2 (20% of steady state GDP) and labor taxes τ_L^w are calibrated at 0.3, as in Eggertsson (2011) .

The system of equations can be simplified into two equations that determine inflation and output in the short run. Units are chosen so that in the steady state, i.e the long run, $Y_L = \xi_L = 1$, implying that $\lambda = (\frac{\theta-1}{\theta})(1-G_L)^{-\tilde{\sigma}^{-1}}(1-\tau_L^w)$. Furthermore, given equations (1)-(5) in the long run $\Pi_L = Y_L = 1$, $1 + i_L = \beta^{-1}, C_L = 1 - G_L$ and

$$K_L = F_L = \frac{(1-\tau_L^w)(1-G_L)^{-\frac{1}{\tilde{\sigma}}}}{1-\alpha\beta} \tag{7}$$

This allows for an explicit characterization of inflation and output in the short-run.

What makes the model particularly tractable is 1) that it is purely forward looking, and 2) the assumption of an absorbing steady state. This implies that (1)-(5) can be combined into an AS and an AD curve that determine Π_S and Y_S. It is assumed that the shock ξ_S is large enough so that the zero bound is binding, i.e., the solution $\Pi_S = 1$ implies negative nominal interest rate. Accordingly, $i_S = 0$.

[11]We choose to specify policy in this way because the policy specified as a Taylor rule accommodates alternate equilibria, as demonstrated by Benhabib, Schmitt-Grohé and Uribe (2001). Our policy specification is an equilibrium selection device where these alternate equilibria do not arise. This particular policy rule may be considered a special case ($\phi_\pi \to \infty$)of the standard Taylor rule of the form:

$$i_t = r_t^e + \phi_\pi \pi_t + \phi_y Y$$

where the alternate equilibria are ignored. Similarly, our results would be identical if we conducted our analysis with the standard Taylor rule as long as $\phi_\pi + \phi_y > 1$ and if the self-fulfilling expectations equilibria are ignored. Our exercise involves a two-state Markov chain, such that the preference shock goes back to zero in the high state. Since there are no internal propagation mechanisms (persistence in Phillips curve or the Taylor rule), the economy would switch to a zero inflation steady state even with $\phi_\pi << \infty$.

[12]As in Eggertsson (2011) we assume that lump-sum taxes adjust to clear the government budget constraint. Accordingly the evolution of government debt is irrelevant due to Ricardian equivalence.

1. **AD curve**: The Euler equation (1) and the Resource Constraint (5) define an AD relationship in the short run:

$$Y_S = G_S + (1 - G_L)\left\{\left[1 - \beta\mu\Pi_S^{-1}\right]\frac{\xi_S}{(1-\mu)\beta}\right\}^{\tilde{\sigma}} \tag{8}$$

2. **AS curve**: The New-Keynesian Phillips curve (2)-(4) and the resource constraint (5), define the AS curve in the short run:

$$\frac{\frac{\theta}{\theta-1}\lambda\xi_S Y_S^{1+\omega} + \alpha\beta(1-\mu)K_L}{\xi_S(1-\tau_S^w)(Y_S - G_S)^{-\frac{1}{\tilde{\sigma}}}Y_S + \alpha\beta\,(1-\mu)F_L}\frac{1-\alpha\beta\mu\Pi_S^{(\theta-1)}}{1-\alpha\beta\mu\Pi_S^{\theta(1+\omega)}} = \left(\frac{1-\alpha\Pi_S^{\theta-1}}{1-\alpha}\right)^{\frac{1+\omega\theta}{1-\theta}} \tag{9}$$

where K_L and F_L are given by (7).

Figure 1: AS-AD representation in the non-linear Calvo

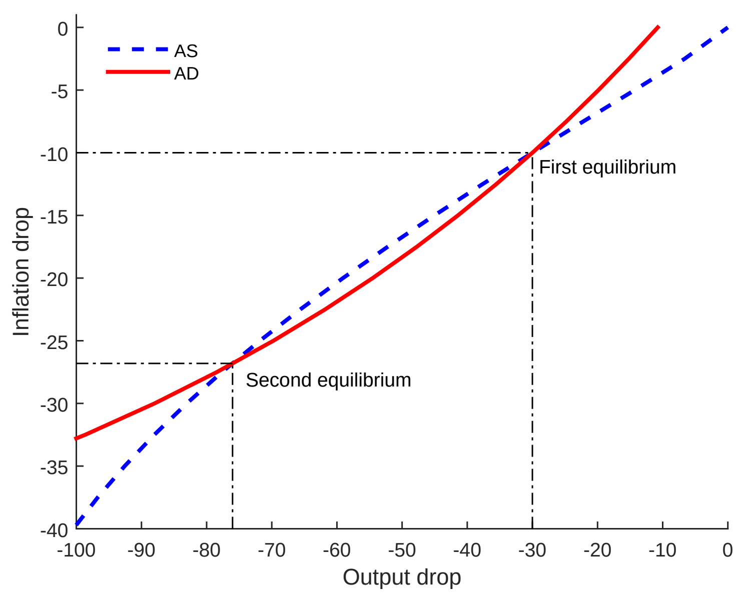


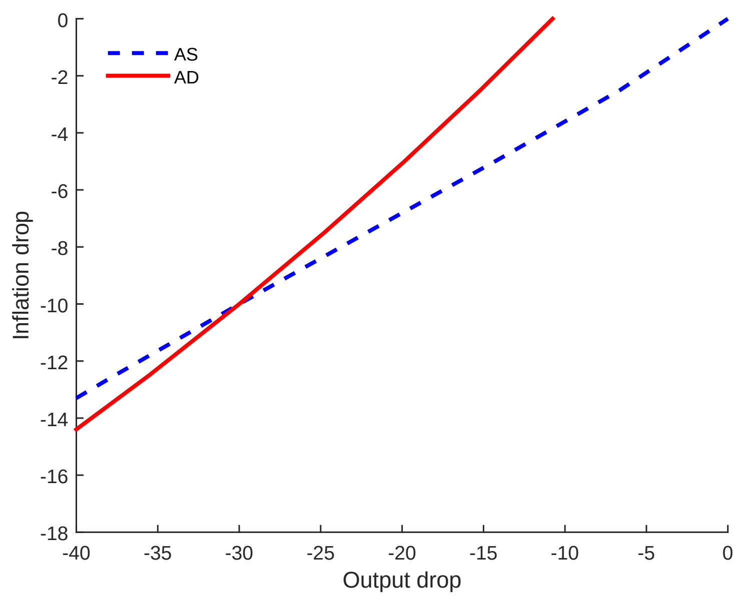
Figure 1 plots up these two curves.[13] Parameters are chosen as in Eggertsson (2011) to deliver a Great Depression scenario of 30% drop in output and 10% drop in inflation where the two curves intersect. The parametrization is further discussed in section 4.

The red (solid) line is the Aggregate Demand curve (8) for the case when the zero lower bound is binding. The dashed blue line is the Aggregate Supply curve (or the New Keynesian Phillips Curve) given by (9). There curves have two intersections. Moving from right to left, they are termed the First and the Second Equilibrium respectively.

[13]These equations are plotted as follows: For a given (fine) grid of numbers for gross inflation, AS and AD are just equations in one variable (output). Find the values of output that satisfy these equations at the grid points. It is straightforward for the AD equation. The AS equation is non-linear and so a non-linear optimization routine in Matlab 'fzero' is used to find the values of output that satisfy the AS equation (starting from some initial value). A web appendix on the authors website provides more details on this algorithm and its robustness. In experiment there has always been found only one positive real value for Y that satisfies the AS equation for a given Π.

The first equilibrium corresponds to a 30% output drop and 10% deflation. The second equilibrium, is a hyperdeflation equilibrium that generates 26% drop in prices per annum and over 75% drop in output. The first equilibrium is very similar to, and exhibits properties of the log-linear approximate equilibrium studied in the literature, such as Eggertsson and Woodford (2003). It is locally unique and determinate. The second equilibrium, however, is locally indeterminate (Blanchard and Kahn 1980). This means that for small perturbation there are infinitely many solutions that satisfy the equilibrium conditions. Moreover, it can be shown that indeterminate equilibrium is also not learnable/E-stable.[14] Accordingly, the next subsection focuses on the determinate equilibria, leaving the analysis of second equilibrium to future work as its economic interpretation is less clear. [15] As shown in Figure 2, the AS and AD curves are "almost linear" up to the stable equilibrium. The next section compares the solution obtained of the non-linear model to the solution from the log-linear approximation. It shows that the key qualitative prediction of the theory obtained from the log-linearized model still applies in the non-linear model.

Figure 2: The stable equilibrium in the non-linear Calvo model



[14]See McCallum (2007) for comparisons of E-Stability and determinacy criterions. Similar conclusions have been arrived by Christiano, Eichenbaum and Johannsen (2018).

[15]Henceforth, the figures display the comparative statics around the first equilibrium which is locally determinate. The web appendix on the authors' website provides matlab codes that run the simulations to establish this indeterminacy.

3 Output drops at the ZLB in the non-linear and the linear model

A key prediction of the log-linearized New Keynesian model is that for small shocks, there can be a large drop in output and inflation. This occurs due to a bifurcation. A natural first question is if this is an artifact of the log-linearization. This section confirms this phenomenon in the nonlinear model. As shown in Eggertsson (2011), the log-linearized model under the assumption of a two state Markov chain with an absorbing state can be expressed as:

1. Aggregate Demand
$$\hat{Y}_S = \hat{G}_S + \frac{\sigma}{(1-\mu)}(\mu\pi_S - r^e_S)$$
where $\hat{Y}_S \equiv \frac{Y_S-Y_L}{Y_L}$; $\hat{G}_S \equiv \frac{G_S-G_L}{Y_L}$; $\pi_S \equiv \Pi_S - 1$; $r^e_S \equiv \log\beta^{-1} + (1-\mu)\log\xi_S$ and $\tilde{\sigma} = \sigma(1-G_L)$

2. Aggregate Supply
$$(1-\beta\mu)\pi_S = \kappa\hat{Y}_S + \kappa\psi\left(\chi^w\hat{\tau}^w_S - \sigma^{-1}\hat{G}_S\right)$$
where$\chi^w \equiv \frac{1}{1-\tau^w_L}$; $\hat{\tau}^w_S \equiv \tau^w_S - \tau^w_L$; $\kappa \equiv \frac{(1-\alpha)(1-\alpha\beta)(\omega+\sigma^{-1})}{\alpha(1+\omega\theta)}$ and $\psi \equiv \frac{1}{\sigma^{-1}+\omega}$

Figure 3: AS-AD: Exact and log-linear approximation

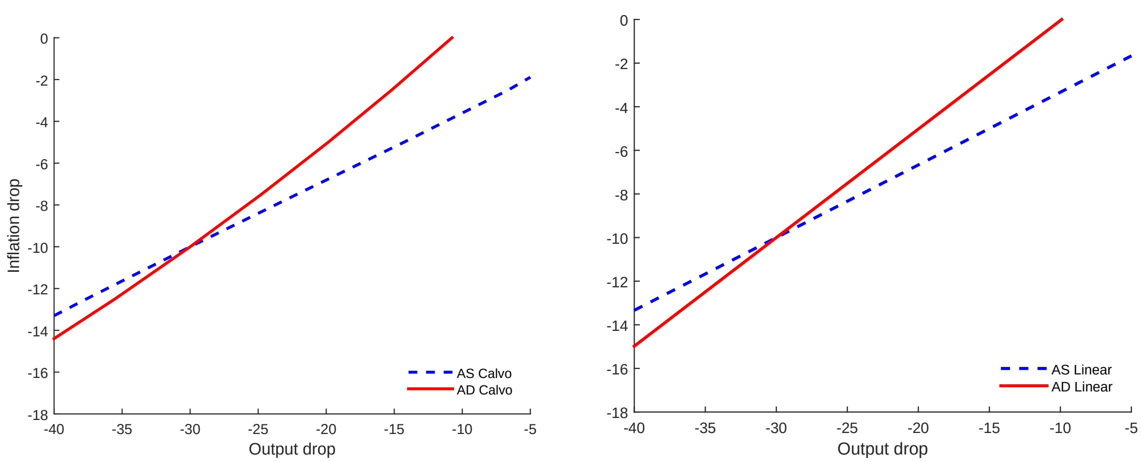


Figure 3 plots the AS and AD curves for the exact non-linear Calvo model and compare the solution to a log-linear approximation. Both the exact model solution and the log-linear approximation are parameterized to generate a 30% drop in output and 10% drop in inflation at the intersection of the curves. The method used to parameterize the model is discussed further in Section 4, this section outlines features of the solution that do not rely on the particular parameterization, even if tying the model down with numbers is useful for illustration.

Let us first consider the log-linear approximation. Proposition 3 in Eggertsson (2011) establishes that the condition for local determinacy in the log-linear approximation of the model at the ZLB is

$$L(\mu) \equiv (1-\mu)(1-\mu\beta) - \kappa\mu\sigma > 0 \tag{10}$$

which says that the AD curve needs to be steeper than the AS curve. This will always be the case for a small enough μ, i.e. if the probability of going back to "long run" is large enough. As the probability of staying in the depressed state increases, the two curves become close to parallel and the drop in output and inflation goes to -∞. The log-linear approximation of the model exhibits an explosion at a critical value $\bar{\mu}$ at which point no solution exists (the lines are parallel). This is called the *bifurcation point*. When $\mu > \bar{\mu}$ there is indeterminacy in the log-linearized model. This explosive behavior is at the heart of the model. It explains how "small" shocks can trigger large drops in output.

The right hand side panel of figure 4 shows an example of the comparative static as μ is increased in the log-linear approximation of the model. This leads to larger output drop and larger deflation. The right hand side panel of figure 5 shows the exact bifurcation point in the log-linear approximation. The second and third rows of the right hand side of of figure 6 shows how output and inflation behave as a function of μ. Output and prices drop without a bound as μ increases converging to $-\infty$ at the bifurcation point, which is not shown in the figure.

Let us now move to the exact non-linear model. Is this bifurcation only an artifact of the log-linearization? Recall that from figure 1 we are considering comparative statics of the first equilibrium, which is well behaved and determinate. In this equilibrium the relative slope of AD and AS are the same in the non-linear solution and the log-linear approximation as shown in 3. Figure 4 shows the effect of increasing the persistence of the shock in the non-linear model (left hand side) similarly leads to a larger recession in the non-linear model. The key difference between the non-linear and the log-linear approximation is show in figure 5. The bifurcation point occurs in the log-linear model (see right hand panel) when the linear AD and AS curves are paralell at $\bar{\mu}$. When $\mu > \bar{\mu}$ then the relative slope of the two lines is reversed, and the model is indeterminate. Meanwhile, we see that in the non-linear model there is also a critical value for two curves when they are tangent to each other as shown in the left hand side panel of figure 5. However, output and inflation do not explode at the bifurcation point. Instead, as seen in figure 6, bifurcation occurs at $1-\mu = 0.093$ where deflation is 16% and the output drop is 48%. As can be seen in figure 5 this happens as the two curves no longer intersect, i.e. the AS curve is convex and is tangent to the AD curve, at finite values of output and inflation.

An interesting implication of this solution is that a marginal increase in μ beyond the bifurcation point implies that the non-linear model has no solution. Hence the indeterminacy region in the log-linearized model does not seem to be of much practical interest; in the non-linear counterpart the comparable region of the parameter space corresponds to non-existence of equilibria.

The bound ([10](#)) on μ is, however, different for the nonlinear model. It is implicitly a condition requiring μ to be small enough so that two curves AD and AS intersect tangentially. If that condition fails to be satisfied there is non-existence rather than indeterminacy. The remainder of the paper focuses on the case in which the model is away from the bifurcation point, so that an equilibrium exists, and analyzes the determinate equilibria in the non-linear model.

Figure 4: AS-AD: Comparative Statics with respect to μ

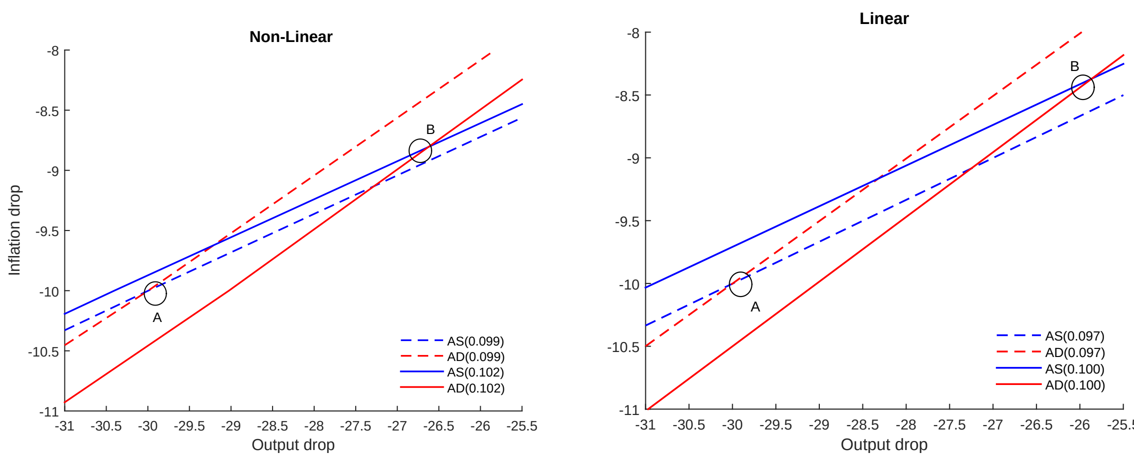
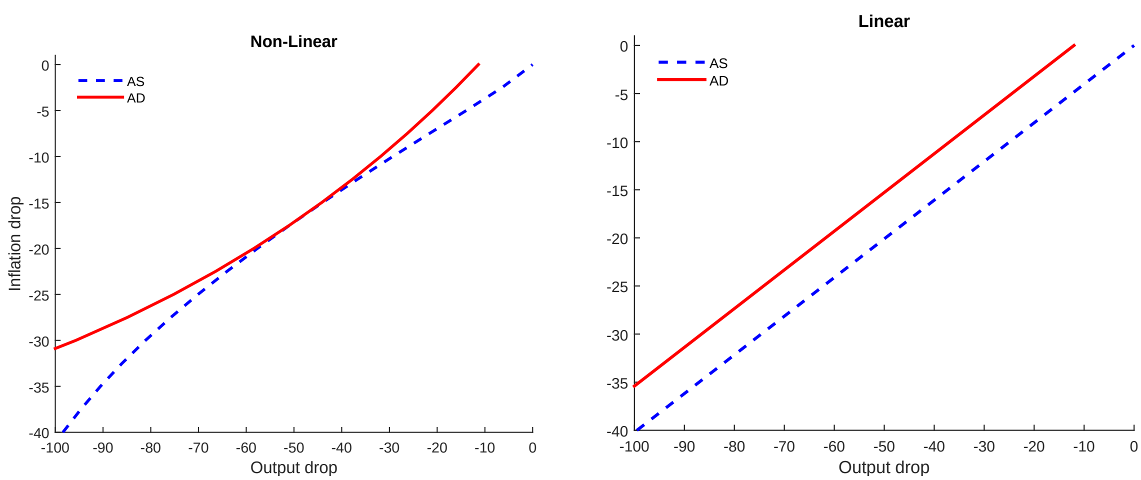


Figure 5: Bifurcation point: AS - AD

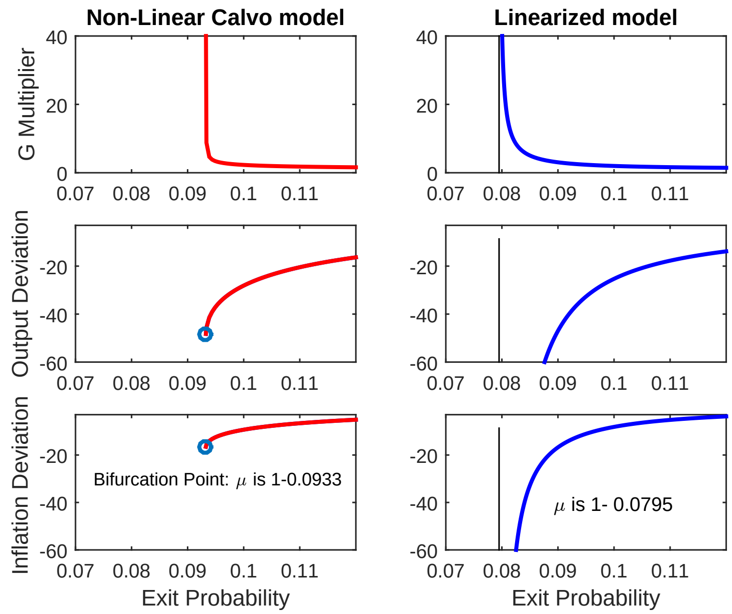


4 Multipliers and paradoxes in numerical experiments

This section first describes the parameterization in more detail for the two scenarios discussed in the introduction, then moves to analyze tax and spending multipliers.

The Great Depression scenario is defined as an event in which output drops by 30% and inflation (annual) by 10%. The Great Recession is defined as a less extreme event with 10%

Figure 6: Bifurcation point: Multipliers versus Exit probability



drop in output and 2% drop in inflation. The parameters and shocks are chosen to maximize the posterior distribution of the model for both the linear and non-linear model. The priors are discussed and method for parameterization is discussed in detail in Denes and Eggertsson (2009) and are reproduced in the Appendix. The key focus of that paper is the sensitivity of the conclusions drawn to the choice of priors. In this paper, we investigate whether the policy conclusions derived from the linear model are robust to considering a non-linear model. The mode of the posterior of the model is reported in Table 2 for the exact model and the log-linear approximation. Observe from Table 2 that the exact choice of parameters and shocks needed to generate the GD and GR will depend on whether the parameterized is based on the exact non-linear model or its log-linear approximation. This, of course, should not be surprising. The interesting question is if, given the procedure for choosing the parameters, it matters if one uses the exact model vs. its log-linear approximation. Does this choice have any effect on the economic inference one draws once these two equilibrium characterizations have been used to replicate numerically a specific economic scenario (GR or GD)? That is the focus of the remainder of this section.[16]

Rows 3 and 4 of Table 2 show the multipliers of government spending and labor tax cuts in the linear and the non-linear models at zero interest rate for the two numerical experiments. In the GD scenario the multiplier is 2.42 in the non-linear model, and 2.22 in the log-linear

[16]The mode is computed using a Matlab maximization routine developed by Christopher Sims. All codes used to construct the tables and the figures in this paper are available on the authors' websites. In Appendices A.5-A.7, we report the priors and the estimated posteriors. The posterior distribution is estimated using a Metropolis algorithm for the non-linear model, following the method detailed in Denes and Eggertsson (2009) for the log-linearized model.

Table 2: Parameterizations under "Great Depression" and "Great Recession' scenarios'

Scenario	Great Depression		Great Recession	
Model	Non-Linear	Linear	Non-Linear	Linear
$\hat{Y}$	-30%	-30%	-10%	-10%
π	-10%	-10%	-2%	-2%
$\frac{\partial \hat{Y}}{\partial G}$	2.4166	2.2168	1.2905	1.1828
$\frac{\partial \hat{Y}}{\partial \tau^w}$	1.5889	0.9953	0.1198	0.1499
α	0.7696	0.7721	0.7752	0.7871
β	0.9969	0.9969	0.9970	0.9970
$\tilde{\sigma}^{-1}$	0.9981	1.4561	1.3498	1.6125
ω	1.3244	1.7465	2.4811	1.7415
θ	10.4066	13.1190	14.6357	13.6012
$1-\mu$	0.09851	0.0965	0.0987	0.1393
r^e_S	-0.0134	-0.0111	-0.0118	-0.0136

approximation. The tax-increase multiplier is 1.59 vs 0.99. Hence both the large value of the government spending multiplier at the ZLB and the positive effects of tax increases at the ZLB (the paradox of toil) are maintained in the non-linear model. All the multipliers are smaller in the GR scenario, but the differences between the linear and non-linear model are of the same order. Figure 7 shows the comparative static of increasing government spending in the linear and non-linear model. Qualitatively the effects are the same, since the relative slopes of the two curves are the same, albeit the exact numbers differ. Even quantitatively, the effect of sizable government spending increases shown in figure 7 are not very different across the two models.[17] Comparative statics for labor tax cut are shown in figure 8, confirming the paradox of toil.[18]

It is instructive to look at the behavior of the multiplier across the two models. The key economic insight of the literature that relied on log-linear approximation was that as the output drop becomes bigger, the government spending multiplier becomes larger. This phenomenon is shown in figure 6. In both models, as the probability of the staying at the zero bound increases, the drop in output intensifies. In the linearized model this means that the multiplier of government spending goes to ∞ as the drop in output goes to -∞. In the non-linear model, however, bifurcation occurs at a finite level of output drop. At that point the multiplier of government spending is very large or about 40.

Figure 9 compares the two models for an increase in price flexibility. Only the AS curve is affected by change in the price flexibility parameter. As prices become more flexible, more firms adjust them downward in response to a liquidity trap shock and hence more severe is the down-

[17] In the figure below, government spending is temporarily increased in both models from 20 percent of steady state output to 30 percent of steady state output

[18] In figure 8 the steady state payroll tax is temporarily reduced from its steady state of 30 percent of GDP to 26 percent of GDP.

Figure 7: Government Spending: Comparative Statics

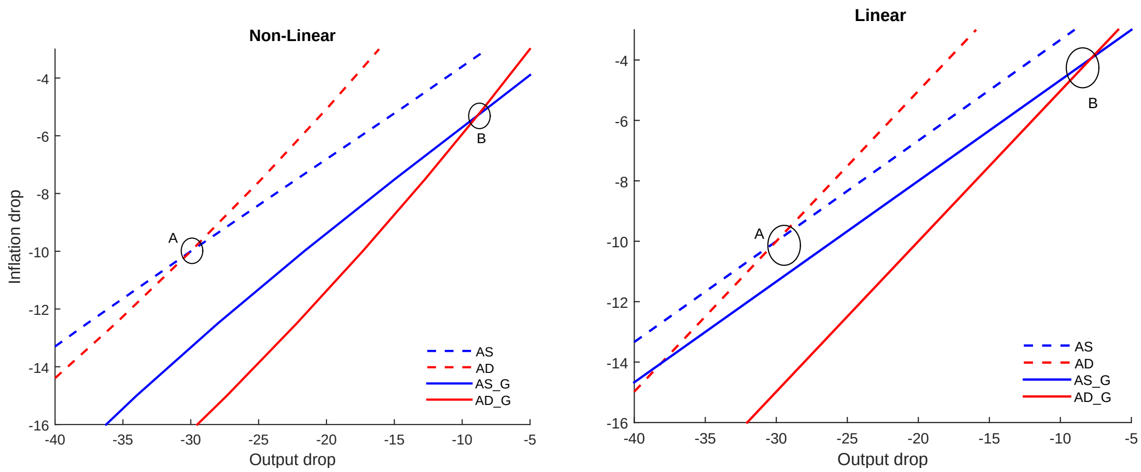
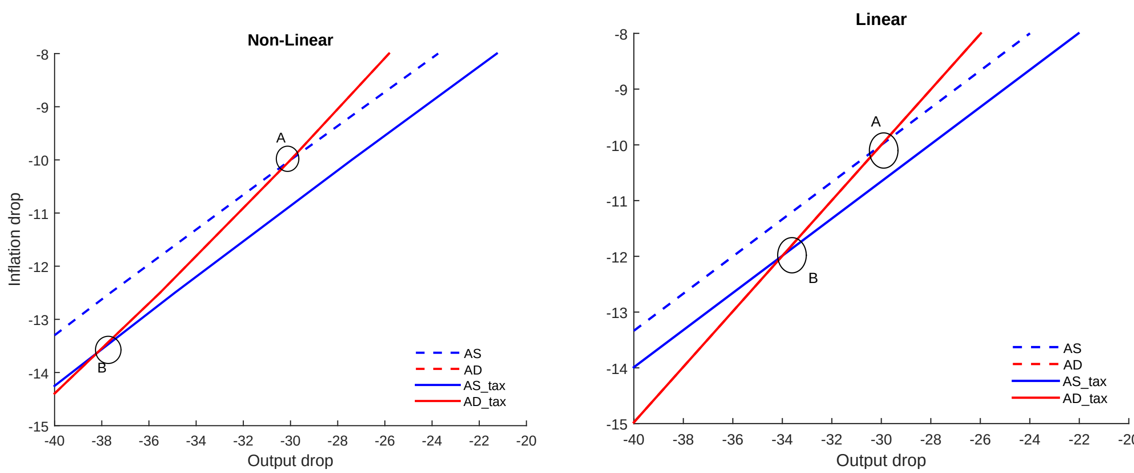


Figure 8: Tax Cut: Comparative Statics

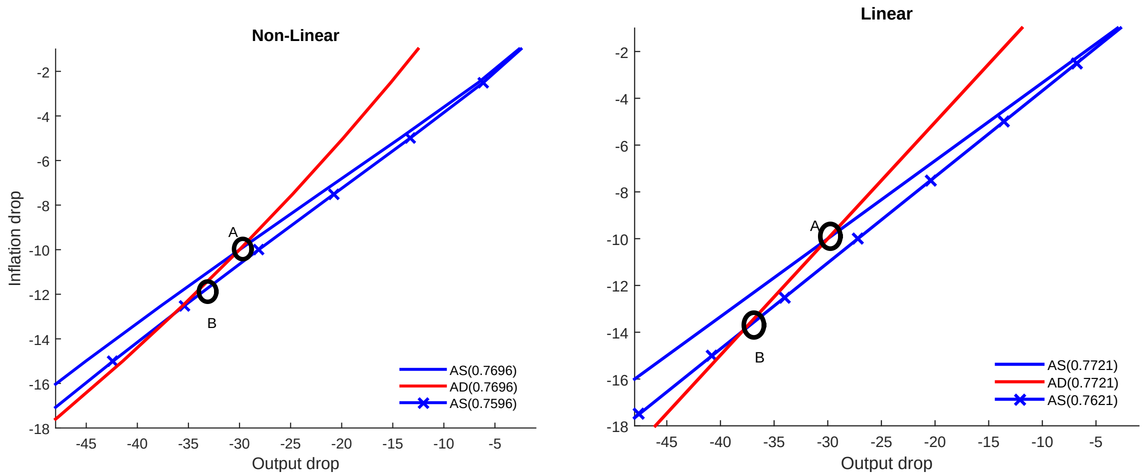


turn. This perverse effect has been extensively explored in Eggertsson (2011) and Christiano, Eichenbaum and Rebelo (2011) among others and is confirmed in the non-linear model. This paradox is discussed in Bhattarai, Eggertsson and Schoenle (2014) in a more general setting away from the ZLB in an estimated DSGE model.

5 Comparison to Rotemberg pricing

We show that the policy disagreements in the non-linear Rotemberg model documented in Boneva, Braun and Waki (2016) and Miao and Ngo (2016) emerge because of implausibly large resource costs spent on price adjustment. There exist, however, appropriate modifications to the non-linear Rotemberg model, often applied in the literature, that yield quantitatively similar policy implications as the Calvo model as we discuss in Section 5.1.

Figure 9: Paradox of Flexibility



Notes: The parentheses show the value of α corresponding to the curves.

In the Rotemberg (1982) model, the intermediate firms make the decision to change prices by incorporating the following cost of adjusting prices into their profit function:

$$\frac{\alpha_r}{2}\left(\frac{P_t(i)}{P_{t-1}(i)}-1\right)^2 Y_t.$$

The log-linearized Rotemberg and Calvo models coincide if the adjustment parameter is related to the Calvo price adjustment probability in the following way:

$$\alpha_r = \frac{\alpha(\theta-1)(1+\omega\theta)}{(1-\alpha)(1-\alpha\beta)}.$$

Following the same algebra as in the Calvo model, the following system of equations for the Rotemberg model can be derived under the same assumption about uncertainty and government policy as before.

1. Euler
$$1 = \beta(1-\mu)\left(\frac{1-G_L}{C_S}\right)^{-1/\tilde{\sigma}}\frac{1}{\xi_S} + \beta\mu\frac{1}{\Pi_S}$$

2. NK Phillips Curve
$$[1-\beta\mu](\Pi_S-1)\Pi_S = \frac{\theta-1}{\alpha_r}\left(\frac{\theta}{\theta-1}\frac{\lambda C_S^{\tilde{\sigma}^{-1}}Y_S^{\omega}}{(1-\tau_S^w)}-1\right)$$

where $\lambda \equiv \frac{\theta-1}{\theta}(1-\tau_w)(1-G_L)^{\tilde{\sigma}^{-1}}$.

3. Resource Constraint

$$Y_S\left[1-\frac{\alpha_r}{2}(\Pi_S-1)^2\right]=C_S+G_S,$$

$$\text{where}\quad \alpha_r=\frac{\alpha(\theta-1)(1+\omega\theta)}{(1-\alpha)(1-\alpha\beta)} \tag{11}$$

This system of equations can be simplified to yield a system of two equations (AD-AS) in two unknowns as before:

1. **AD curve**: The AD curve in the low state is derived from the Euler equation and the Resource Constraint, together with the stipulation that the ZLB is binding:

$$Y_S=\left[1-\frac{\alpha_r}{2}(\Pi_S-1)^2\right]^{-1}\left\{G_S+(1-G_L)\left\{\left[1-\beta\mu\Pi_S^{-1}\right]\frac{\xi_S}{(1-\mu)\beta}\right\}^{\tilde{\sigma}}\right\}$$

2. **AS curve**: The AS curve in the low state is derived from the New-Keynesian Phillips curve and the resource constraint:

$$[1-\beta\mu](\Pi_S-1)\Pi_S=\frac{\theta-1}{\alpha_r}\left(\frac{\theta}{\theta-1}\frac{\lambda\{Y_S\left[1-\frac{\alpha_r}{2}(\Pi_S-1)^2\right]-G_S\}^{\tilde{\sigma}^{-1}}Y_S^{\omega}}{(1-\tau_S^{w})}-1\right)$$

Figure 10: Non-linear Rotemberg model vs its log-linearized approximation

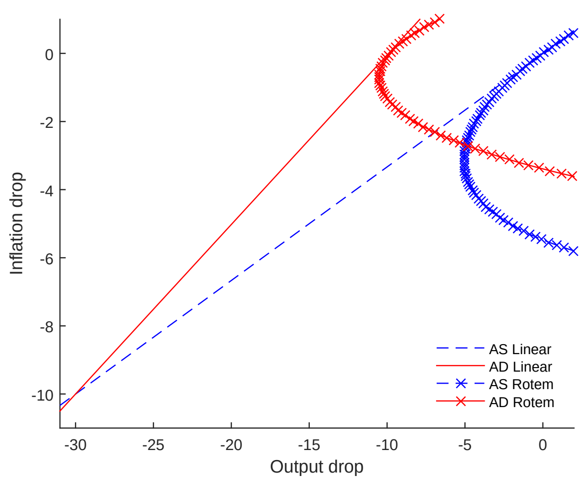


Figure 11: Resource Costs in the Rotemberg model

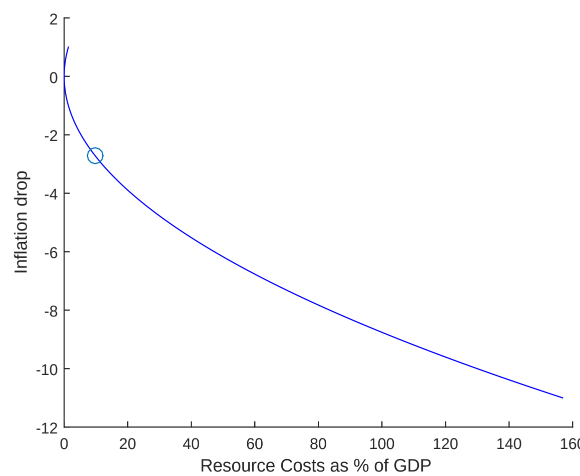


Figure 10 plots the AS and the AD curves for the non-linear Rotemberg model and compares to the log-linear approximation assuming the following parameters: the log-linear Rotemberg model and the log-linear Calvo model coincide. Accordingly the the same parameterization is used for the log-linear Rotemberg model as in Table 2, using equation 11 to back out the relevant parameters for the log-linearized Rotemberg model. The figure shows that the log-linear Rotemberg model generates the Great Depression scenario, consistent with 30 percent drop in output and 10 percent drop in inflation. The non-linear Rotemberg model, however, cannot generate the GD scenario. To illustrate why this is the case, the same parameterization is fed into the non-linear model and shown in the figure 10. It is evident, from the graph, that a fundamental divergence occurs between the linear and non-linear model.

As the figure reveals, the nonlinear Rotemberg model behaves in a fundamentally different way compared to the log-linear approximation. While the log-linear model generates a GD scenario, the non-linear Rotemberg model show a different equilibrium altogether. The two curves intersect at inflation drop of 2.70% and output drop of 4.95%. More importantly, however, the AD curve has changed its slope. If inflation goes below approximately -1% then aggregate demand becomes upward sloping, i.e., the larger the fall in the prices, the more output is demanded in the aggregate. This is in contrast to the non-linear and the log-linear approximation of the Calvo model discussed so far, as well as different from the log-linear approximation of the Rotemberg model.

Is this behavior of the non-linear Rotemberg model economically meaningful? Figure 11 sheds some light on this. It shows that as inflation goes down, more of the economy's resources go into changing prices. For example, a 10% deflation as observed during the Great Depression would require over 100% of the output of the economy to change prices. Clearly then, by assumption, the Rotemberg model parameterized with physical resource costs of this kind cannot generate a Great Depression, a result also reported in Boneva, Braun and Waki (2016). More generally, the large implied resource costs of changing prices is key to understanding why the AD curve changes its slope. It is instructive to write out the aggregate resource constraint

$$Y_S = C_S + G_S + Y_S \frac{\alpha_r}{2} (\Pi_S - 1)^2$$

The three components of aggregate spending, Y_S, are private consumption, government consumption and resources spent to change prices. In the conventional analysis, demand goes down at the ZLB because a reduction in inflation (dis-inflation) increases the real interest rate. This reduces private consumption, C_S, and thus aggregate demand Y_S. In this model, however, there is one additional force. As deflation increases, each firm "demands" more output to change prices. At relatively low values of inflation, this force is weak, and thus the aggregate demand slope remains downward. Figure 10 shows that this happens until about -1% inflation in our example. This effect is absent in the non-linear Calvo model. In fact, AD curve in that case is almost linear in inflation. Hence the major difference in the results between the two models is the implication that the large resource costs of price adjustment have on aggregate demand. The AS curve also changes slope in the Rotemberg model with high enough deflation, i.e. output increases as inflation goes down. The reason is once again the high resource costs of changing prices. As inflation goes into negative territory, more resources go into changing prices. This reduces the consumption of the representative agent, thus increasing his marginal utility of income. This, in turn, will increase his labor supply, and increase aggregate supply.

The large resource costs of price changes rely heavily on the quadratic form of price adjustment and the fact that the value of α_r is very large.[19] It seems implausible that at moderate levels of inflation, the actual output needed to change prices increases quadratically, corresponding to over 100% of all output. Indeed, since the costs are quadratic, then inflation of about -10% is not technically feasible in the numerical experiment reported above as it would overwhelm the workforce. Since this particular mechanism is driving most of the non-linearities in the Rotemberg model it does not seem to be a good price-setting assumption to study extreme events of this form.[20]

There is, however, a simple modification of the Rotemberg model in which case the results from this model are similar to the Calvo model. Rotemberg's (1982) preferred interpretation of the quadratic costs faced by firms was not physical menu costs, but instead that "... *there is a cost that captures the negative effect of price changes, particularly price increases on the reputation of firms*". An alternative interpretation of the Rotemberg model is thus simply that these are costs perceived by firms, and are taken into account only in the firm maximization problem, but do not correspond to direct cost of price changes. In this case, the model simplifies to

$$Y_S = \left\{ G_S + (1 - G_L) \left\{ \left[1 - \beta\mu\Pi_S^{-1}\right] \frac{\xi_S}{(1-\mu)\beta} \right\}^{\tilde{\sigma}} \right\}$$

$$[1 - \beta\mu](\Pi_S - 1)\Pi_S = \frac{\theta - 1}{\alpha_r} \left(\frac{\theta}{\theta - 1} \frac{\lambda\{Y_S - G_S\}^{\tilde{\sigma}^{-1}} Y_S^{\omega}}{(1 - \tau_S^{w})} - 1 \right)$$

Figure 12 compares the non-linear ('modified Rotemberg') model to the linearized model. The modified Rotemberg model is parameterized to match the GD scenario.[21] As the figure shows, the non-linear model has no difficulty replicating this scenario, and the qualitative features of the model are now more in line with the non-linear and log-linearized Calvo model, eliminating the

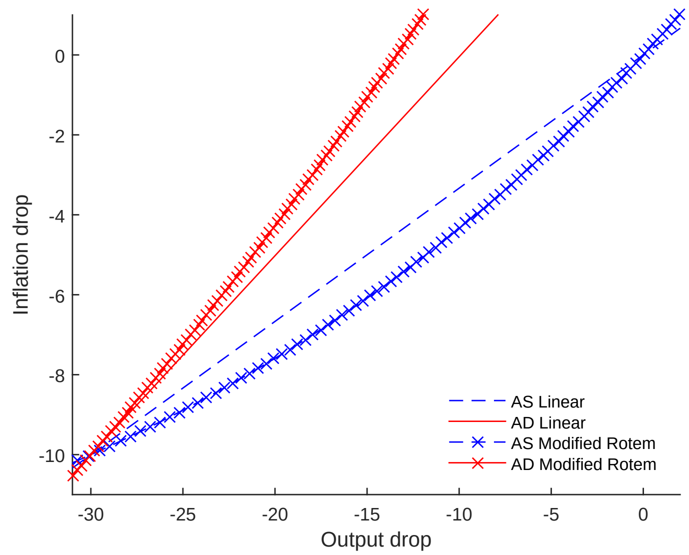
[19] An anonymous referee pointed out the relevance of firm-specific labor markets in making α_r high.This was shown in detail by Keen and Wang (2007). They show that the assumption of firm-specific labor markets requires a larger α_r compared to economy wide labor markets, given the structural parameters from the Calvo model. We thank an anonymous referee for making this connection. Miao and Ngo (2016) show that even in a model with economy-wide labor markets the log-linear and the non-linear solutions of the Rotemberg model disagree because of implausibly large resource costs. Another approach would be to calibrate a very low α_r.

[20] Table 9 in the Appendix reports the multipliers in the Rotemberg model along with the resource costs.

[21] Table 10 in the Appendix reports the calibrated parameters and the corresponding mutlipliers in the modified Rotemberg model and contrast them with those of the log-linearized model.

different implication the two models had for the conduct of policy.

Figure 12: Modified Rotemberg Model versus the Linear Model



5.1 Modifications to the Rotemberg model

Ascari and Rossi (2012) have already suggested the mathematical variation of the modified Rotemberg model suggested in last subsection. In their model, while there are resource costs of changing prices borne by firms, these costs are rebated to the household lump sum. Accordingly, the costs do not show up in the economy's resource constraint. In our example, however, this interpretation would be a stretch, as it would imply lump-sum taxation above 100 percent of GDP. We prefer, therefore, the more reduced form psychological interpretation where no lump-sum taxation is required. Alternatively, the researcher could model these costs directly in the utility function as Benhabib, Schmitt-Grohé and Uribe (2001).[22]

A complementary approach is to abandon the somewhat unattractive assumption of quadratic costs of price adjustment, following the business cycle literature on investment adjustment costs. It has been recognized in the business cycle literature that quadratic costs of investment adjustment can lead to non-sensical conclusions for the same reason as here, large enough drop in investment lead to implausible adjustment costs. Christiano and Davis (2006, p. 13-14) resolve this by fitting a tenth degree polynomial, with standard Chebyshev interpolation that replicates the first order investment dynamics implied by a quadratic adjustment function – yet at the same time implies plausible equilibrium resource adjustment costs. The same approach can be used

[22] Another approach, suggested by an anonymous referee, would be to consider the price-adjustment costs as intermediate goods and thus define output net of intermediate goods, i.e. $Y_t = C_t + G_t$. This would also yield similar qualitative results across the log-linear and the non-linear solutions.

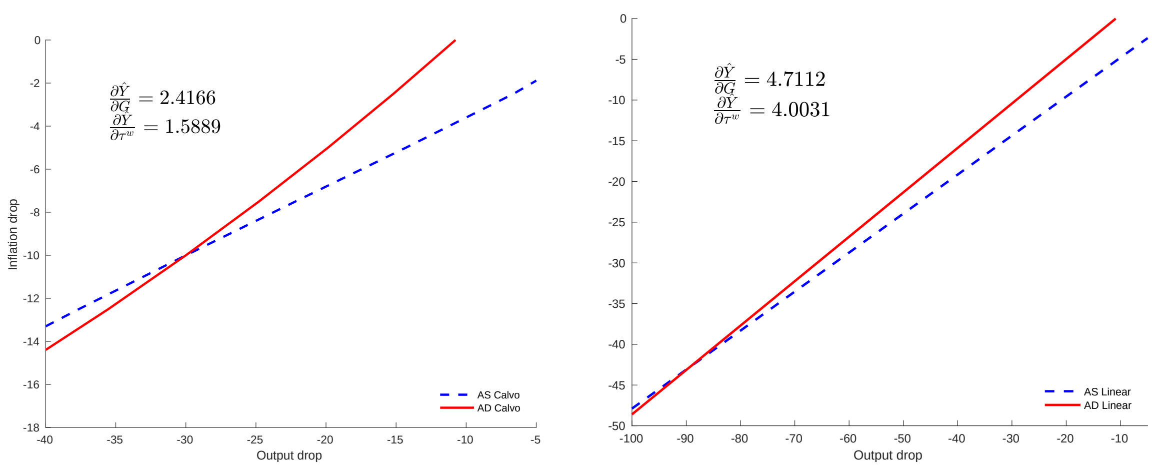
here, but this is left for future research.[23]

The key takeaway from this section is not that there is a serious policy disagreement between the Rotemberg and the Calvo model. Instead, a particular variant of Rotemberg model with significant output costs of price adjustment generates economically meaningless dynamics when solved non-linearly. It is recommended that the applied researcher exercises caution while deriving policy implications from such a model, without appropriate modifications.

6 Discussion: Extensions and Limitations

6.1 Interpreting nonlinearities

Figure 13: AS-AD: Exact and log-linear approximation using the non-linear parameterization



We calibrated the non-linear and the linearized Calvo solutions to generate identical scenarios in terms of output drop and deflation. This choice of parameters and shocks is made to contemplate a "policy scenario" of the form we think is of economic interest. This does not mean, however, that the two solution methods generate the same outcome in general, if we assume the same parameters and shocks. Recall, that the shocks and parameters were in fact chosen so as to generate the particular policy scenario in question. To make that clear, we show in Figure 13, the non-linear and the linear model for the same set of parameters (Column 2 of table 2) that generate the Great Depression scenario in the non-linear model.[24] These parameters were chosen by estimating the non-linear model. Clearly, the linearized model generates extreme results in

[23] As these authors point out the assumption of the adjustment function being quadratic is arbitrary, and has usually been only made for simplicity in the context of non-linear models that are ultimately approximated by log-linear approximation. Taken literally, the non-linear counterpart then implies non-sensical investment adjustment costs.

[24] We thank an anonymous referee for stressing this point to us and suggesting this numerical example.

this example and would yield very different policy conslustions. This, however, is not a general result. In some parameter configurations, the non-linear model leads to more extreme results.

This highlights that the interpretation of our experiments is not that non-linearities are "irrelevant" in the sense that if one picks a particular set of parameters and shocks then it does not matter if the model is solved using log-linear approximation or non-linear solution methods. Instead, the interpretation is that if the parameters and shocks are chosen so as to match a particular economic scenario, then it is of little consequence if the researchers uses a log-linear approximation or considers the non-linear model, at least for the special case we consider here. In the Appendix A.8, we show that what is of primary importance in our example is the choice of the underlying shocks, rather than the particular parameter values. We use identical parameter values across the two solution methods but calibrate the shocks to replicate a common economic scenario. The policy multipliers across the solution methods are essentially identical.

What is one to make of this? One interpretation, and takeaway, is that the non-linear model represents the truth, while the log-linear approximation represents a model mis-specification. If one adopts this interpretation the log-linearized model is a less ideal procedure than parameterizing and estimating the fully non-linear model.

Another interpretation is that the first order dynamics captured by the the log-linear approximation may be closer to the "truth" in the sense that they apply for a broad class of model, e.g. both the Rotemberg and the Calvo model follow these first order dynamics. Thus, the researcher might hope that this representation is less misspecified than either of the non-linear micro-founded models for the issue under study and feel more confident about conclusion that did not depend upon second order dynamics. For example, few researchers would seriously argue that the cost of price changes are convex as in Rotemberg, or that the opportunity for changing prices arrives independently of the past price setting as in Calvo. Instead, the hope in both cases is that the underlying assumptions provide a representation that is not too far from the truth, or in any event capture the first order dynamics.[25] Nevertheless, it may very well be that the first order dynamics of the model are still approximately accurate in a more complete micro-founded variation of the model that generates more plausible dynamics of price dispersion.

Questions about model specification are important issues for further research, for which we do not offer any concrete answers. Instead, our goal is more limited and simply address the question: If one uses a consistent way of parameterizing the Calvo model at the ZLB, is it im-

[25] This is, in fact, why Rotemberg costs are typically assumed to be quadratic. It is not because a quadratic function is a plausible description of reality, but instead because it implies analytically convenient linear first order dynamics. Another reason to be more interested in the first order properties of the model for extreme shocks, rather than the non-linear counterpart, is that price dispersion becomes extremely large in that case to an extent that seems implausible. Moreover, this does not seem in sync with recent evidence as in Nakamura et al. (2016)

portant which solution method is adopted for the question at hand? This paper is a simple but widely studied example in which case the answer to this question is no. This should serve as a warning for researchers that use estimates for log-linearized model and apply those parameters to the non-linear solution. What is needed in the latter case is a full re-estimation of the model.

Our experiment also provides a counterexample to the claim by Judd, Maliar and Maliar (2017, JMM henceforth) that approximation errors in the log-linearized version of the standard New Keynesian model "are so huge that even under most optimistic scenario makes these numerical solutions inacceptable for any application." TO make this assessment, JMM compare the performance of a log-linear approximation of Smets and Wouters (2007) model to its non-linear counterpart under particular calibrations for the deep parameters of the model and the structural shocks. One key point of this paper, however, is that any calibration of the deep parameters of the model (the Smets-Wouters model is an alternative example) as well as the shock processes, are always chosen *conditional* on a particular approximation method in order to match the underlying data. Hence, in order to compare a fully non-linear solution to a log-linear approximate solution of the Smets-Wouters model, the more informative comparison would be to parameterize each solution to match exactly the same data. Once this is done, it becomes interesting to ask if the two approximation methods generate meaningful differences for key policy questions that they are aimed to answer, since this is in fact the process by which these models are used in practice (the modeler estimates the model with the data, and then does experiments to inform policy decisions). As we have seen in the case of a simple ZLB experiment, a log-linear approximation would fail miserably according to JMM's criterion. Yet, it generates essentially the same quantitative conclusion for the key policy questions, even under extreme scenarios meant to match the Great Depression. It is yet to be seen if the same insight applies to the Smets-Wouters model.

6.2 Estimation with a cost-push shock

It has been suggested that a source of missing deflation in the US economy during the Great Recession can be attributed to the presence of adverse technology and oil-price shocks. To consider the robustness of our result to this feature we re-compute the mode by also introducing an aggregate supply shock ϱ_S in the linear and non-linear Calvo models. ϱ_S follows the two-state Markov chain and has the same prior distribution as the demand shock. We replicate the Great Recession scenario, to hit 10% drop in output and 2% drop in inflation. Table 8 reports the mode and multipliers for the log-linear and the non-linear solutions of the Calvo model. The estimation does put considerable weight to the cost-push shocks across both solutions, as this allows the model to capture both output and inflation dynamics with a price rigidity that is closer to the prior. Since a cost-push shock during ZLB is inflationary, the multiplier on government spending is expected to be smaller, which is in fact true for both the solution concepts. Quantitatively, the log-linear and the non-linear solutions generate similar fiscal multipliers even in the presence of a cost-push shock.

7 Conclusion

This paper studies the non-linear properties of the Calvo model at the ZLB. It finds no policy disagreement between the non-linear and the log-linear solutions of the model. It also suggests that the economics behind policy disagreements stemming from certain versions of the Rotemberg model are unrealistic. A modified Rotemberg model – arguably more in spirit with Rotemberg's original suggestion – delivers similar result as the Calvo model.

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A Appendix

A.1 System of equations for the Non-linear Calvo model under two-state Markov chain assumption

1. Euler
$$1 = \beta(1-\mu)\left(\frac{C_L}{C_S}\right)^{-1/\tilde{\sigma}} \frac{1}{\xi_S}\frac{1}{\Pi_L} + \beta\mu\frac{1}{\Pi_S}$$
where $C_L = 1 - G_L$, and $\Pi_L = 1$.

2. NK Phillips Curve
$$K_S = \frac{\theta}{\theta-1}\lambda\xi_S Y_S^{1+\omega} + \alpha\beta(1-\mu)K_L + \alpha\beta\mu\Pi_S^{\theta(1+\omega)}K_S$$
$$F_S = \xi_L(1-\tau_S^w)C_S^{-\frac{1}{\tilde{\sigma}}}Y_S + \alpha\beta\ (1-\mu)F_L + \alpha\beta\mu\Pi_S^{(\theta-1)}F_S$$
$$\frac{K_S}{FS} = \left(\frac{1-\alpha\Pi_S^{\theta-1}}{1-\alpha}\right)^{\frac{1+\omega\theta}{1-\theta}}$$
where $\lambda \equiv \frac{\theta-1}{\theta}(1-\tau_w)(1-G_L)^{\tilde{\sigma}^{-1}}$.

3. Monetary Policy Rule:
$$i_t = i_{zlb} \text{ ,in the short run}$$

4. Resource Constraint
$$Y_t = C_t + G_t$$

5. Fiscal Policy rule: Normal Government Spending G_L is calibrated at 0.2 (20% of steady state GDP). Labor Taxes τ_L^w. are calibrated at 0.3. To calculate fiscal multipliers: when ZLB is binding, $G_S > G_L$. For Paradox of toil, $\tau_S^w < \tau_L^w$.

6. Preference Shock
$$\xi_S < \xi_L \equiv 1$$

A.2 *Mapping the shock in the linear Calvo with the non-linear Calvo*

$$\hat{r}_S^e = \log\beta^{-1} + (1-\mu)\log\xi_S$$
$$\implies \log\xi_S = \frac{(r_S^e - \log\beta^{-1})}{(1-\mu)}$$
$$\implies \xi_S = \exp(\frac{(r_S^e - \log\beta^{-1})}{(1-\mu)})$$

Defines a correspondance between ξ_S *and* $\hat{r}_S^e a$

A.3 Determinacy in the non-linear model

Under the two state assumption (with a binding zlb), the dynamic system can be written as:

1. Euler

$$0 = \beta\mu\mathbb{E}_t^S\left[\left(\frac{Y_{t+1}^S - G^S}{Y_t^S - G^S}\right)^{-1/\tilde{\sigma}}\frac{1}{\Pi_{t+1}^S}\right] + \beta(1-\mu)\left[\left(\frac{1}{Y_t^S - G^S}\right)^{-1/\tilde{\sigma}}\frac{1}{\xi^S}\right] - 1 \qquad (A.1)$$
$$\equiv f^1(\ \mathbb{E}_t^S Y_{t+1}^S, \mathbb{E}_t^S \Pi_{t+1}^S, Y_t^S)$$

2. NK Phillips Curve

$$0 = \frac{\theta}{\theta-1}\lambda\xi^S(Y_t^S)^{1+\omega} + \alpha\beta(1-\mu)K^L + \alpha\beta\mu\mathbb{E}_t^S\left[(\Pi_{t+1}^S)^{\theta(1+\omega)}K_{t+1}^S\right] - K_t^S \qquad (A.2)$$
$$\equiv f^2(\mathbb{E}_t^S\Pi_{t+1}^S, \mathbb{E}_t^S K_{t+1}^S, Y_t^S, K_t^S)$$

$$0 = \xi^S(1-\tau_t^w)(Y_t^S - G^S)^{-\frac{1}{\tilde{\sigma}}}Y_t^S + \alpha_c\beta(1-\mu)F^L + \alpha\beta\mu\mathbb{E}_t^S\left[(\Pi_{t+1}^S)^{(\theta-1)}F_{t+1}^S\right] - F_t^S \qquad (A.3)$$
$$\equiv f^3(\mathbb{E}_t^S\Pi_{t+1}^S, \mathbb{E}_t^S F_{t+1}^S, Y_t^S, F_t^S)$$

$$0 = \frac{K_t^S}{F_t^S} - \left(\frac{1-\alpha(\Pi_t^S)^{\theta-1}}{1-\alpha}\right)^{\frac{1+\omega\theta}{1-\theta}} \equiv f^4(\Pi_t^S, K_t^S, F_t^S) \qquad (A.4)$$

having substituted in the resource constraint and assuming that $G_{t+1}^S = G_t^S = G^S$. Observe that the expectation of each variable conditional on that the shock will remain in the low "short-run" state in the next period is denoted by $\mathbb{E}^S$. Writing the system in this way, this represents a regular rational expectation system that can be solved via standard methods, e.g. using Blanchard and Kahn (1980).

Linearizing this system of equations yields:

$$\left[\frac{\partial f^1}{\partial \mathbb{E}_t^S Y_{t+1}^S}\right]_{ss}\mathbb{E}^S\hat{Y}_{t+1}^S + \left[\frac{\partial f^1}{\partial Y_t^S}\right]_{ss}\hat{Y}_t^S + \left[\frac{\partial f^1}{\partial \mathbb{E}_t^S\Pi_{t+1}^S}\right]_{ss}\mathbb{E}_t^S\pi_{t+1}^S = 0 \qquad (A.5)$$

$$\left[\frac{\partial f^2}{\partial K_t^S}\right]_{ss}k_t^S + \left[\frac{\partial f^2}{\partial Y_t^S}\right]_{ss}\hat{Y}_t^S + \left[\frac{\partial f^2}{\partial \Pi_t^S}\right]_{ss}\pi_t^S + \left[\frac{\partial f^2}{\partial \mathbb{E}_t^S K_{t+1}^S}\right]_{ss}\mathbb{E}^S\hat{K}_{t+2}^S = 0 \qquad (A.6)$$

$$\left[\frac{\partial f^3}{\partial F_t^S}\right]\hat{F}_t^S + \left[\frac{\partial f^3}{\partial Y_t^S}\right]_{ss}\hat{Y}_t^S + \left[\frac{\partial f^3}{\partial \Pi_t^S}\right]_{ss}\pi_t^S + \left[\frac{\partial f^3}{\partial \mathbb{E}_t^S F_{t+1}^S}\right]_{ss}\mathbb{E}_t^S\hat{F}_{t+1}^S = 0 \qquad (A.7)$$

$$\left[\frac{\partial f^4}{\partial K_t^S}\right]_{ss}\hat{K}_t^S + \left[\frac{\partial f^4}{\partial F_t^S}\right]_{ss}\hat{F}_t^S + \left[\frac{\partial f^4}{\partial \Pi_t^S}\right]_{ss}\pi_t^S = 0 \qquad (A.8)$$

where small letters $\{\ \hat{Y}_t^S, \pi_t^S, \hat{K}_t^S, \hat{F}_t^S\}$ indicate deviations of the variables from the solution around which the equations are linearized. This system of equations is determinate if there are at least 3 unstable eigenvalues outside of the unit circle. Numerically simulations (using reds and solds) suggest that the second equilibrium fails to meet the determinacy conditions and hence is locally unstable. Henceforth, the paper only investigates the properties of the stable equilibrium (in this case, it is the one that produces 30% drop in output and 10% drop in inflation).

A.4 Determinacy in the linear model - Proposition 3 from Eggertsson (2011)

Lemma 1. *An approximate equilibrium defined by the collection of stochastic processes* $\{\hat{y}_S, \pi_S\}$ *at zero interest rates is a locally unique bounded equilibrium, for a given path of these variables after the ZLB stops binding and a given value of* r_S^e*. if C1 and C2 hold, where*

$$r_S^e < -\Gamma_{\tau^w}\hat{\tau_S^w} - \Gamma_G \hat{g}_S \tag{C1}$$

$$L(\mu) \equiv (1-\mu)(1-\mu\beta) - \kappa\mu\sigma > 0 \tag{C2}$$

A.5 Priors used for likelihood estimation

The same priors are used across all likelihood maximization procedures. These were first used by Denes and Eggertsson (2009) and the consequent literature. Table 3 reproduces them.

Table 3: Distributions of Priors

	distribution	mean	standard deviation	Prior 5%	Prior 50%	Prior 95%
α	beta	0.66	0.05	0.5757	0.6612	0.7402
β	beta	0.99669	0.001	0.9949	0.9968	0.9981
$1-\mu$	beta	1/12	0.05	0.0198	0.074	0.1788
$\tilde{\sigma}^{-1}, \sigma^{-1}$	gamma	2	0.5	1.2545	1.9585	2.8871
ω	gamma	1	0.75	0.1519	0.82	2.4631
θ	gamma	8	3	3.7817	7.6283	13.4871
$-r_S^e$	gamma	0.010247	0.005	0.0036	0.0094	0.0196

Notes: $\sigma(1-G_L) = \tilde{\sigma}$, where G_L is calibrated to 0.2.

A.6 Posteriors in log-linear model

Table 4: Posteriors for the structural parameters and the shocks in the log-linear Calvo model

		Great Recession				Great Depression			
	distribution	Pos 5%	Pos 50%	Pos 95%	Mode	Pos 5%	Pos 50%	Pos 95%	Mode
α	beta	0.7149	0.7731	0.8227	0.7871	0.6876	0.7616	0.8121	0.7721
β	beta	0.9948	0.9968	0.9982	0.997	0.9949	0.9969	0.9982	0.9969
$1-\mu$	beta	0.1003	0.1557	0.2361	0.1393	0.0739	0.1027	0.1425	0.0965
σ^{-1}	gamma	0.8426	1.2995	1.8834	1.29	0.8792	1.2435	1.6906	1.1649
ω	gamma	0.9383	2.0761	4.0045	1.7415	0.7648	1.8224	3.6873	1.7465
θ	gamma	8.4754	13.8292	20.5524	13.6012	9.753	14.2748	22.6658	13.119
$-r_S^e$	gamma	0.0262	0.0159	0.0083	0.0136	0.0277	0.0149	0.0082	0.0111

Table 5: The posterior distribution of the fiscal multipliers in the log-linear Calvo model

	Great Recession				Great Depression			
	Pos 5%	Pos 50%	Pos 95%	Mode	Pos 5%	Pos 50%	Pos 95%	Mode
$\frac{\partial \hat{Y}}{\partial \tau^w}$	0.2809	0.1099	0.0519	0.1499	1.6222	0.7086	0.305	0.9953
$\frac{\partial \hat{Y}}{\partial G}$	1.0775	1.1599	1.3401	1.1828	1.4451	1.8627	2.7951	2.2168

A.7 Posteriors in non-linear Calvo model

Table 6: Posteriors for the structural parameters and the shocks in the non-linear Calvo model

		Great Recession				Great Depression			
	distribution	Pos 5%	Pos 50%	Pos 95%	Mode	Pos 5%	Pos 50%	Pos 95%	Mode
α	beta	0.7083	0.7589	0.8205	0.783	0.7594	0.7684	0.7765	0.7696
β	beta	0.9949	0.9968	0.9981	0.997	0.9948	0.9967	0.9982	0.997
$1-\mu$	beta	0.0662	0.0841	0.0986	0.0939	0.0991	0.1085	0.1197	0.0985
$\tilde{\sigma}^{-1}$	gamma	1.1338	1.6101	2.0967	1.3498	0.9712	1.0419	1.1281	0.9981
ω	gamma	1.7784	3.124	4.9063	2.4506	1.2407	1.3207	1.4043	1.3244
θ	gamma	10.9097	16.7474	22.7434	15.7444	9.5527	10.2092	10.5979	10.4066
$-r^e_S$	gamma	0.0189	0.012	0.0073	0.0115	0.0282	0.0198	0.0131	0.0134

Notes: $\sigma(1-G_L)=\tilde{\sigma}$, where G_L is calibrated to 0.2.

Table 7: The posterior distribution of the fiscal multipliers in the non-linear Calvo model

	Great Recession				Great Depression			
	Pos 5%	Pos 50%	Pos 95%	Mode	Pos 5%	Pos 50%	Pos 95%	Mode
$\frac{\partial \hat{Y}}{\partial \tau^w}$	0.1855	0.0915	0.0522	0.1088	1.6866	0.9463	0.5781	1.5889
$\frac{\partial \hat{Y}}{\partial G}$	1.2017	1.2824	1.4208	1.2677	1.7167	1.9586	2.4295	2.4166

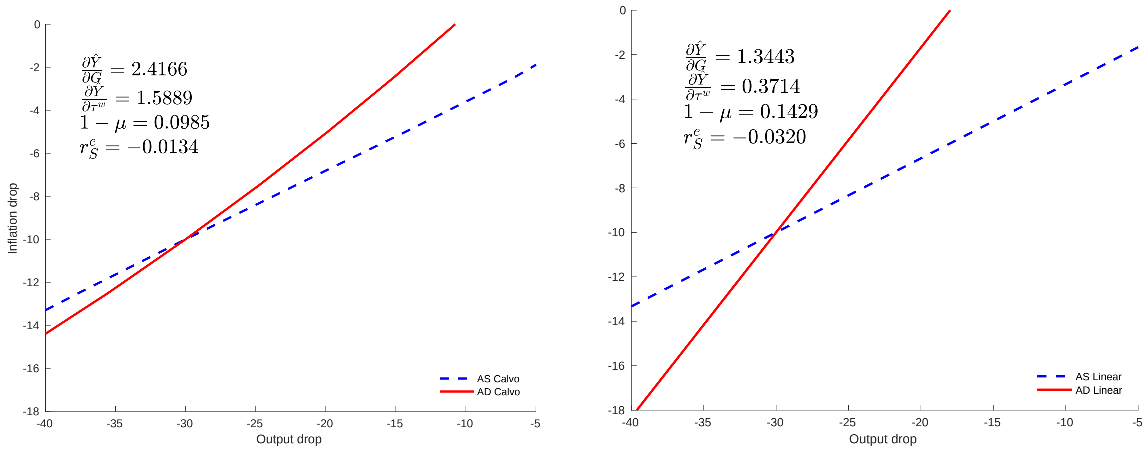
A.8 Using a common non-linear Calvo parametrization

To provide further evidence on similarities between the comparative statics in the log-linear and nonlinear Calvo models, we plot the AS and the AD curves in the two models using a common parametrization. The structural parameters are such that the non-linear model reproduces the Great Depression sccenario. These are reported in Column 2 of Table 2. In order to generate an identical scenario in the log-linear model, we recalibrated the persistence of the ZLB shock μ and the severity of the shock r^e_S such that the log-linear model also generates the Great Depression scenario i.e. output drop ($\hat{y}_S$) is 30% and inflation drop ($4\hat{\pi}_S$) is 10%.

$$\mu=\beta^{-1}\left(1-\frac{\kappa\hat{y}_S}{\hat{\pi}_S}\right);\quad r^e_S=\sigma^{-1}\left((1-\mu)\hat{y}_S-\sigma\mu\hat{\pi}_S\right)$$

where $\sigma(1-G_L)=\tilde{\sigma}$, and G_L is calibrated to 0.2. Figure 14 compares the two variants for this parametrization. The output drop and inflation drop is same across the two models. The magnitude of fiscal multipliers in the log-linear model is less than the non-linear model. Contrast this with the results reported in figure 13 where the log-linear model produces much higher multipliers, because of high persistence of the ZLB shock.

Figure 14: AS-AD: Exact and log-linear approximation using the non-linear parameterization



A.9 Modeling AS Shifter

We model a cost-push shock in the log-linear and the non-linear Calvo model to show robustness of our results to this extension. We assume that the markup shock follows a two-state process identical to the preference shock. In the log-linear model, the AS curve is modified to:

$$(1 - \beta\mu)\hat{\pi}_S = \kappa\hat{y}_S + \kappa\psi\hat{\lambda}_S^p; \quad \hat{\lambda}_t = \log(1 + \lambda^p) - \log(1 + \lambda^p); \quad 1 + \lambda^p = \frac{\theta}{\theta - 1}$$

The non-linear NK Phillips Curve is modified to include the markup shock θ_S

$$K_S = \frac{\theta_S}{\theta_S - 1}\lambda\xi_S Y_S^{1+\omega} + \alpha\beta(1-\mu)K_L + \alpha\beta\mu\Pi_S^{\theta_S(1+\omega)}K_S$$

$$F_S = \xi_L(1 - \tau_S^w)C_S^{-\frac{1}{\tilde{\sigma}}}Y_S + \alpha\beta\ (1-\mu)F_L + \alpha\beta\mu\Pi_S^{(\theta_S - 1)}F_S$$

$$\frac{K_S}{FS} = \left(\frac{1 - \alpha\Pi_S^{\theta_S - 1}}{1 - \alpha}\right)^{\frac{1+\omega\theta_S}{1-\theta_S}}$$

where $\lambda \equiv \frac{\theta_L - 1}{\theta_L}(1 - \tau_w)(1 - G_L)^{\tilde{\sigma}^{-1}}$.

The relationship between parameters θ_S and $\hat{\lambda}_S^p$ is as follows:

$$\theta_S = \frac{\exp(\hat{\lambda}_S^p + \log\frac{\theta}{\theta - 1})}{\exp(\hat{\lambda}_S^p + \log\frac{\theta}{\theta - 1}) - 1}$$

The prior distribution for $\hat{\lambda}_S^p$ is identical to prior for $-r_S^e$. Further we assume that markup shock is perfectly correlated with the preference shock. Table 8 displays the estimated posterior mode for each model.

Table 8: Great Recession parameterizations for the nonlinear Calvo and the Linearized Model

Scenario	with AS shifter		Benchmark	
Model	Non-Linear	Linear	Non-Linear	Linear
$\frac{\partial \hat{Y}}{\partial G}$	1.2686	1.0850	1.2905	1.1828
$\frac{\partial \hat{Y}}{\partial \tau^w}$	0.1521	0.0851	0.1198	0.1499
α	0.7663	0.7946	0.7752	0.7871
β	0.9967	0.9970	0.9970	0.9970
$\tilde{\sigma}^{-1}$	0.8652	2.1849	1.3498	1.6125
ω	1.7428	1.4274	2.4811	1.7415
θ_L	12.6791	13.9012	14.6357	13.6012
$1-\mu$	0.1572	0.1563	0.0987	0.1393
r^e_S	-0.0130	-0.0231	-0.0118	-0.0136
$\hat{\lambda}^p_S$	0.0067	0.0085	n/a	n/a

A.10 Multipliers in the Rotemberg Model

Table 9 reports the multipliers in the Rotemberg model and compares them with those obtained in the Modified Rotemberg model. Notes below the table explain in detail the parametrization used. The core point is that if there is physical resource costs of changing prices (e.g. menu costs) that take on a quadratic form, then the non-linear model is unable to produce a significant drop in output and inflation.

Table 9: Multipliers in the Modified Rotemberg versus the Rotemberg Model

Scenario	Great Depression		Great Recession	
Model	Rotemberg	Mod. Rotem.	Rotemberg	Mod. Rotem.
$\hat{Y}$	-1.5%	-30%	-5.2%	-10%
π	-4.6%	-10%	-1.5%	-2%
$\frac{\partial \hat{Y}}{\partial G}$	0.4107	1.7684	0.6977	1.2455
$\frac{\partial \hat{Y}}{\partial \tau^w}$	-0.5542	0.2589	-0.2363	0.0835

[a] The parameters used in calculating these numbers are reported in Columns 1 & 3 of table 10. They match the Great Depression and the Great Recession Scenarios for the Modified Rotemberg Model.

A.11 Parameters and Multipliers in the Modified Rotemberg Model

Table 10: Parameterizations for the Modified Rotemberg and the Linearized Model

Scenario	Great Depression		Great Recession	
Model	Mod. Rotem.	Linear	Mod. Rotem.	Linear
$\hat{Y}$	-30%	-30%	-10%	-10%
π	-10%	-10%	-2%	-2%
$\frac{\partial \hat{Y}}{\partial G}$	1.7684	2.2168	1.2455	1.1828
$\frac{\partial \hat{Y}}{\partial \tau^w}$	0.2589	0.9953	0.0835	0.1499
α	0.7720	0.7721	0.7667	0.7871
β	0.9957	0.9969	0.9970	0.9970
$\tilde{\sigma}^{-1}$	1.2496	1.4561	1.1849	1.6125
ω	1.7109	1.7465	2.2316	1.7415
θ	10.4726	13.1190	14.4678	13.6012
$1-\mu$	0.0910	0.0965	0.1326	0.1393
r^e_S	-0.0193	-0.0111	-0.0155	-0.0136

α reported here is the Calvo parameter : 1 - probability of price adjustment. We estimate this α in our likelihood maximization in order to use the same priors across all the specifications. It is straightforward to back out the Rotemberg α_r using the formula reported in the main text.