Incomplete Markets and Exchange Rates

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Our paper: domestic $\times$ intn'l market incompleteness matters for exchange rates

This paper

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* analytical heterogeneity ala Constantinides & Duffie (1996)
* integrated financial markets (with “many" assets)
* presence of uninsurable idiosyncratic risk $\Longrightarrow$ wedge in the aggregate Euler eqn

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Resolve Backus-Smith puzzle if and only if

exchange rates are risky

- domestic risk goes down when exchange rate depreciates

Closed form solutions

two-country factor model Cox, Ingersoll, & Ross (1985) with heterogenous consumers

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international + domestic markets are incomplete

Exchange Rates:

Corsetti, Dedola and Leduc (2008), Benigno and Thoenissen (2008), Benigno and Küçük (2012), Lustig and Verdelhan (2019)

Heterogeneous Agents:

Constantinides and Duffie (1996), Ramchand (1999), Leduc (2002), Kocherlakota Pistaferri (2007), Krueger & Lustig (2010), Werning (2015),...

Other Explanations:

Itskhokhi Mukhin (2021), Chernov Haddad Itskhoki (2024), Jiang Krishnamurthy Lustig (2024), Kekre Lenel (2024)

Roadmap

1. Representative agent asset pricing model
2. Heterogenous consumers model
3. Testable empirical conditions
4. Conclusion

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$$\mathbb{E}_t[M_{t+1}] = 1/R_{t+1}, \qquad \mathbb{E}_t[M_{t+1}^*] = 1/R_{t+1}^* \qquad \text{(Trade in dom. bonds)}$$

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What does incompleteness mean?

Under complete markets (domestic and international):
$\rightarrow$ Backus Foresi Telmer (2001): Given M_t and observed $\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}$, $\exists$ unique M_t^*.

When international markets are incomplete:
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Assuming time-separable, CRRA preferences, classical int'l macro models assume:

$$\eta_{t+1} = log \underbrace{\left(\frac{P_{t+1}}{P_t}\frac{P_t^*}{P_{t+1}^*}\right)}_{\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}} - log \underbrace{\left(\frac{C_t}{C_{t+1}}\frac{C_{t+1}^*}{C_t^*}\right)^s}_{\frac{M_{t+1}}{M_{t+1}^*}}$$

IM wedge

Int'l trade in bonds disciplines IM wedge [Lustig and Verdelhan (2019)]:

$$\mathbb{E}_t[\eta_{t+1}] = \frac{1}{2} var_t(\eta_{t+1}) - cov_t(m_{t+1}, \eta_{t+1}), \quad \text{(H bond traded)}$$

$$-\mathbb{E}_t[\eta_{t+1}] = \frac{1}{2} var_t(\eta_{t+1}) + cov_t(m^*_{t+1}, \eta_{t+1}) \quad \text{(F bond traded)}$$

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Backus Smith Impossibility with Rep Agent SDF

Assume SDFs, allocations and prices are jointly log-normal 4 Eulers $\Longrightarrow$

$$cov_t(m^*_{t+1} - m_{t+1}, \Delta e_{t+1}) = var_t(\Delta e_{t+1})$$

Lustig & Verdelhan (2019)

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Heterogeneous Consumers, Integrated Markets 1/2

2-country C-CAPM with heterogeneity [Constantinides and Duffie (1996)]:

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-s}, \quad \Delta c_{t+1} = w_{t+1} \sim \mathcal{N}(\mu_{C_t}, \sigma^2_{C_t}),$$

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$$\Delta c^i_{t+1} = \log \left(\frac{\delta^i_{t+1}}{\delta^i_t} \frac{C_{t+1}}{C_t} \right) \sim \mathcal{N}(\mu_{c^i_t}, \sigma^2_{i,t})$$

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Individual consumption draw related to aggregate:

$$\int_i \delta^i_t \, di = 1, \quad \log(\delta^i_{t+1}/\delta^i_t) \sim \mathcal{N}(-\tfrac{y_{t+1}}{2}, y_{t+1})$$

where y is the variance of log of idiosyncratic consumption risk.

- CD96 choose endowment processes to support a no trade (autarky) equilibrium

Heterogeneous Consumers, Integrated Markets 2/2

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analogously for Foreign bond

Aggregate Euler with a Wedge

Individual Euler holds

$$\mathbb{E}_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-s}\left(\frac{\delta^i_{t+1}}{\delta^i_t}\right)^{-s}\right]=\frac{1}{R_t}$$

With the CD'96 income share process $\left[\frac{\delta^i_{t+1}}{\delta^i_t}=\exp(\xi^i_{t+1}\sqrt{y}_{t+1}-y_{t+1}/2))\right]$

$$\mathbb{E}_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-s}e^{\frac{s(s+1)}{2}y_{t+1}}\right]=\frac{1}{R_t}$$

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$$\mathbb{E}_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-s}\underbrace{e^{\frac{s(s+1)}{2}y_{t+1}}}_{e^{\tilde{y}_{t+1}}}\right]=\frac{1}{R_t}$$

$\tilde{y}_{t+1}\equiv\frac{s(s+1)}{2}y_{t+1}$ denotes the log of the variance of the idiosyncratic permanent shocks

pricing of risks with domestic incompleteness

when markets are domestically incomplete: M_t is not generically unique.

$$\mathbb{E}_t \left[\underbrace{\beta \left(\frac{C_{t+1}}{C_t} \right)^{-s} e^{\tilde{y}_{t+1}}}_{\text{``rep-agent"SDF}} \right] = \frac{1}{R_t}$$

$\exists$ "rep-agent" SDF such that het agents agree on pricing of aggregate risks.

internationally complete markets (but domestic incomplete) defined on

$$\Delta e_t = m_t^* - \underbrace{[m_t + \tilde{y}_t]}_{\text{``rep-agent" SDF}}$$

Comovement with Heterogeneous Consumers

The model with H & F traded bonds and heterogeneous Home consumers delivers $cov_t(m^*_{t+1} - \log(\int_i e^{\Delta c^i_t} di)^{-s}, \Delta e_{t+1}) < 0$ *if and only if:*

$$1 \geq -\rho_{\tilde{y}_{t+1},\Delta e_{t+1}} \geq \frac{\sigma_t(\Delta e_{t+1})}{\sigma_t(\tilde{y}_{t+1})}$$

where $\rho_{\tilde{y}_{t+1},\Delta e_{t+1}} \equiv \dfrac{cov_t(\tilde{y}_{t+1}, \Delta e_{t+1})}{\sigma_t(\Delta e_{t+1})\sigma_t(\tilde{y}_{t+1})}$

- Idiosyncratic risk which co-moves with FX recovers a non-traded component
  - F bonds are a poor hedge if $\rho_{\tilde{y}_{t+1},\Delta e_{t+1}} < 0$

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- Idiosyncratic risk which co-moves with FX recovers a non-traded component
  - F bonds are a poor hedge if $\rho_{\tilde{y}_{t+1},\Delta e_{t+1}} < 0$
- relaxed when only one bond intn'lly traded
- preserved when adding trade in more assets

Cox-Ingersoll Ross for Closed form

Let a common factor z_t drive aggregate consumption globally

$$z_{t+1} = \rho z_t + u_{t+1}$$

Home and Foreign aggregate consumption

$$\Delta c_{t+1} = \sqrt{z_t} u_{t+1}; \quad \Delta c^*_{t+1} = \xi^* \sqrt{z_t} u_{t+1}, \quad \xi^* > 1$$

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SDF:

$$m_{t+1} = \log \beta - \gamma \Delta c_{t+1}; \quad m^*_{t+1} = \log \beta - \gamma^* \Delta c^*_{t+1};$$

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uninsurable idiosyncratic risk process

$$\tilde{y}_{t+1} = \tilde{\alpha}_y + \tilde{\phi} \sqrt{z_t} u_{t+1},$$

$\tilde{\phi} \equiv \phi \frac{\gamma(\gamma+1)}{2}$: the cyclicality of idiosyncratic risk

Equilibrium Incomplete Markets Wedge

In the model with heterogeneous consumers, with H & F bonds traded, the incomplete markets wedge is given by:

$$\eta_{t+1} = -\frac{1}{2}\left[(\gamma - \gamma^* - \phi)^2 - \kappa\right] + \left[-\phi - \sqrt{(\phi - \gamma - \gamma^*)^2 - \lambda}\right]\sqrt{z_t}u_{t+1} - \left[\sqrt{\lambda - \kappa}\right]\sqrt{z_t}\epsilon_{t+1},$$

where $\lambda \geq \kappa$ is a parameter governing cross border spanning and

$$\kappa = (\gamma - \gamma^* - \phi)^2 - (\gamma - \gamma^* - \phi)\sqrt{(\gamma - \gamma^* - \phi)^2 - \lambda} \geq 0$$

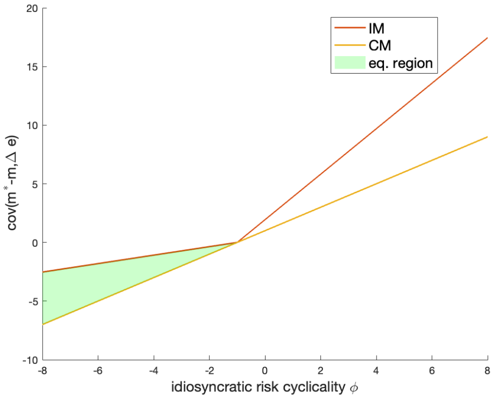
where κz_t is the exchange rate volatility.

The model with heterogeneous consumers, with H & F bonds traded, delivers $cov_t(m^*_{t+1} - m_{t+1}, \Delta e_{t+1}) < 0$ *if and only if* $var(r_{t+1}) < var(r^*_{t+1})$.

* specifically, when $\phi < \gamma - \gamma^* < 0$

Exchange rate cyclicality: Illustration

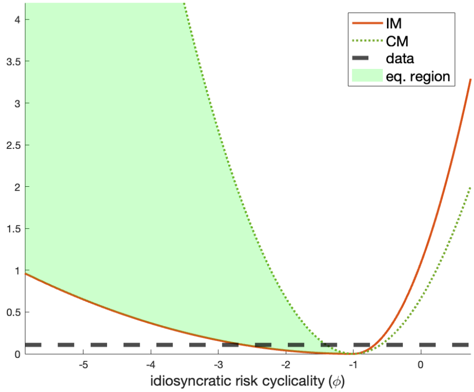
Figure: Backus-Smith Covariance



Shaded regions capture the combinations of ϕ and moments of exchange rates admissible under incomplete markets for $\gamma = 1, \xi^* = 2$.

Exchange rate volatility: Illustration

Figure: Exchange Rate Volatility



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Exchange rate volatility: International Markets

If income risk is counter-cyclical ($\phi < 0$)*, exchange rate volatility is always higher under international complete markets* $\kappa^{CM} > \kappa^{IM}$.

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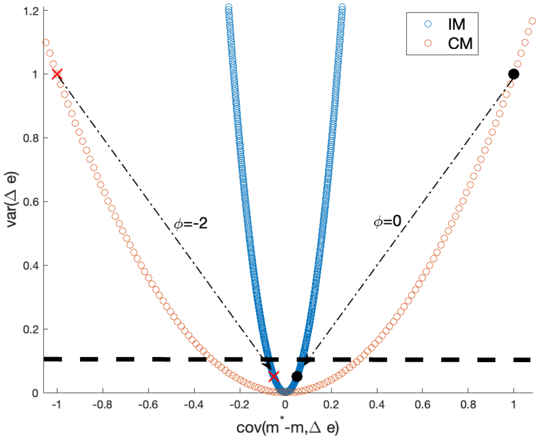

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Testable Empirical Condition

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Data over 1979Q1–2013Q1

* bilateral real exchange rate of U.S. dollar against seven advanced economies.
* estimate of income risk volatility (Bayer, Luetticke,Pham-Dao,& Tjaden 2019)

$$s = 10; \quad \sigma(\Delta e_{t+1}) = 0.11; \quad \sigma(\tilde{y}_{t+1}) = 4.4 \implies RHS = 0.025$$

The measured correlation: $-\rho_{\tilde{y}_{t+1},\Delta e_{t+1}} = 0.05$ substantially higher than needed

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Heterogeneous consumers with uninsurable risk offers a plausible mechanism for exchange rates