The financial origins of non-fundamental risk*

Sushant Acharya† Keshav Dogra‡ Sanjay R Singh§

Abstract

We formalize the idea that the financial sector can be a source of non-fundamental risk. Households' desire to hedge against price volatility can generate price volatility in equilibrium, even absent fundamental risk. Fearing that asset prices may fall, risk-averse households demand safe assets from leveraged intermediaries, whose issuance of safe assets exposes the economy to self-fulfilling fire sales. Policy can eliminate non-fundamental risk by (i) increasing the supply of publicly backed safe assets, through issuing government debt or bailing out intermediaries, or (ii) reducing the demand for safe assets, through social insurance or by acting as a *market maker of last resort*.

Keywords: safe assets, self-fulfilling asset market crashes, liquidity, fire sales
JEL codes: D52, D84, E62, G10, G12

*The authors thank Anatoli Segura (discussant), Toni Ahnert, Ana Fostel, Jess Benhabib, Javier Bianchi, Saki Bigio, Nicolas Caramp, Andrés Carvajal, Edouard Challe, Russell Cooper, Eduardo Dávila, Maryam Farboodi, Jean-Sébastien Fontaine, John Geanakoplos, Piero Gottardi, Frederic Malherbe, Daniel Neuhann, Guillermo Ordoñez, Jacob Robbins, Alp Simsek, Shengxing Zhang and seminar participants at the 17th Annual Cowles Conference on GE, 2022 Financial Stability Conference at the Cleveland Fed, the Barcelona Summer Forum, SNDE 2022, the EUI and UC Davis for comments and feedback. The views expressed in this paper are those of the authors and do not necessarily represent those of the Bank of Canada, the Federal Reserve Banks of New York, San Francisco or the Federal Reserve System.

†Bank of Canada, 234 Wellington St. W, Ottawa, ON K1A 0G9, **Email:** sacharya@bankofcanada.ca
‡FRB New York, 33 Liberty Street, New York, NY 10045, United States, **Email:** keshav.dogra@ny.frb.org
§Federal Reserve Bank of San Francisco and UC Davis **Email:** sjrsingh@ucdavis.edu

It is sometimes argued that the financial sector can itself be a source of risk rather than a means to manage fundamental risk in the real economy (Danielsson and Shin, 2003; Rajan, 2005). We formalize this idea in a model where there are no fundamental shocks, but non-fundamental price volatility can emerge when financial intermediation is permitted. The key mechanism involves a mutual feedback between the risk of a fall in asset prices and investors' purchase of insurance against this risk from financial intermediaries. The fear of a price decline is what motivates investors to demand insurance against such a fall. But it is only because intermediaries sell this insurance to investors that a self-fulfilling fall in prices can occur, as intermediaries are forced to sell assets in order to meet their obligations in the event of a price decline.

We study a two-period rational expectations model with three types of agents: risk-averse households, risk-neutral financial intermediaries and outside non-specialists (Shleifer and Vishny, 1997). There are two perishable goods, which we refer to as cookies and apples. Agents trade physical assets at date 0 ("trees") which yield apples at date 1. Households are endowed with trees and cookies at date 0 but only derive utility from consuming cookies at both dates. Thus, while households utilize trees as a store of value between dates 0 and 1, they are potentially exposed to the fluctuations in the price of trees at date 1. If households are pessimistic and fear that trees will fall in price, they demand insurance against this risk. Financial intermediaries meet this demand, selling state-contingent securities which are backed by their holdings of trees and which pay off in the event of a fall in the price of trees at date 1.

We show that a self-fulfilling asset price crash can emerge in such an environment even absent fundamental risk. In the event of a fall in prices of trees at date 1, intermediaries are forced to sell their holdings of trees to non-specialists in order to pay out on the insurance contracts they sold to households. This fire sale makes the fall in the price of trees self-fulfilling. From the perspective of date 0, it is precisely the possibility of such self-fulfilling fire sales at date 1 which leads households to demand insurance at date 0 in the first place. If households did not anticipate price declines at date 1, they would not demand insurance, intermediaries would never be forced into fire sales and no price declines would occur in equilibrium. Conversely, if intermediaries were not permitted to sell such securities, self-fulfilling fire sales would not occur and households would have no need for insurance to begin with. In this sense, the financial sector can be a source of risk for the rest of the economy.

Our model is highly stylized and includes ingredients that are standard in recent macro-finance models (e.g., Stein 2012) used to study financial stability. In our baseline model, the privately issued safe assets take the form of insurance contracts that guarantee to pay the difference between the strike price and the spot price when asset prices fall. These can be interpreted as out-of-the-money put options purchased by investors from market makers and other financial intermediaries. We show that self-fulfilling fluctuations also occur if intermediaries issue non-state-contingent debt, which can be interpreted as repo contracts. The key element is that the intermediaries selling safe assets, whatever form these take, use the proceeds to take leveraged positions in "risky" assets which expose them to risk of fire sales. These positions resemble trading strategies that often precede episodes of asset price volatility.[1] In this sense, our framework highlights how the private provision of "safe" assets can result in financial market fragility and non-fundamental asset price fluctuations. The supply of safe assets creates its own demand – a *"Say's Law for risk"* – whether these safe assets take the form of insurance contracts, options or risk-free bonds.

Since it is an increase in the supply of private safe assets which opens the door to financial fragility, policy can prevent fragility by reducing the excess return to private safe asset creation and discouraging its supply. First, the government can crowd out the provision of private safe assets by issuing public safe assets: non-state-contingent debt backed by purchases of trees. Households purchase safe assets from the government, rather than from intermediaries, preventing the build-up of intermediary leverage which would have allowed fire sales to occur. The government is never forced to sell at fire sale prices at date 1 because financial intermediaries are net buyers at this date (rather than net sellers, as in the fire sales scenario). Interestingly, a commitment to bailout financial intermediaries following a fall in prices has the same effect: intuitively, a portion of the safe assets issued by the intermediaries becomes "publicly backed" and crowds out unbacked private safe assets. Rather than crowd out private safe assets, a policymaker can also just reduce the demand for these assets, by providing social insurance to households or by acting as a *market maker of last resort* (Buiter and Sibert, 2008), standing ready to buy trees at a fixed discount to their fundamental value. These policies eliminate the possibility of fire sales, even though intervention is never required on equilibrium. The commitment to make transfers to households following a price decline, or to mitigate a price decline by buying assets, reduces households' exposure to a fall in prices and lowers their demand for insurance, preventing the buildup of intermediary leverage which allows fire sales to arise.

[1] See, for example, "How Repo Agreements Juiced Securitized-Debt Leverage" at https://www.bloomberg.com/news/articles/2020-04-15/how-repo-agreements-juiced-securitized-debt-leverage-quicktake for March 2020 meltdown in financial markets. More recently, contracts for differences and total return swap contracts have been linked to the Archegos' meltdown. See, for example, "Billions in Secret Derivatives at Center of Archegos Blowup" at https://www.bloombergquint.com/markets/billions-in-secretive-derivatives-at-center-of-archegos-blowup.

Literature Review Perhaps the paper closest to ours is Bowman and Faust (1997), who showed that the addition of an option market can introduce sunspot equilibria which do not exist without these markets, even when the original economy features complete markets absent sunspots. They stress that options differ from Arrow securities in that they have price-contingent payoffs, and argue that this is why option markets can create new sources of risk, rather than completing incomplete markets or having no effect when the market is already complete.[2] Hens (2000) showed an example where trading of sunspot contingent assets introduced sunspot equilibria. These papers relate to a question posed by Mas-Colell (1992): Can sunspot equilibria exist when underlying economies – absent trading of sunspot-contingent contracts – have unique equilibria? In our economy, instead, private safe asset creation can give rise to non-fundamental price volatility even when these assets do *not* have price-contingent or sunspot-contingent payoffs. While option markets can introduce non-fundamental price volatility, the addition of a market for non-contingent risk-free bonds can also have the same effect. In this sense, our model supports Rajan (2005)'s conclusion that financial innovation could lead to fragility (see also Gennaioli et al. 2012, Simsek 2013 and Gu et al. 2020).

Our analysis also relates to the literature studying pecuniary externalities in models with financial frictions (Lorenzoni, 2008; Bianchi, 2011; Stein, 2012; Dávila and Korinek, 2018).[3] As in much of this literature, our environment features fire sales which emerge because private agents do not internalize the effect of their borrowing on prices in an incomplete markets economy.[4] Interestingly, while much of this literature finds that state-contingent contracts can eliminate financial crises (Krishnamurthy, 2003; Lorenzoni, 2008), in our setting, the *introduction* of state-contingent assets can give rise to crises.[5] Another important difference is that most of this literature studies how financial frictions can amplify fundamental shocks, whereas we show how they can give rise to non-fundamental volatility. In this sense, another related paper is Schmitt-Grohé and Uribe (2020), who study multiple equilibria in a small open economy model. Besides the fact that our environment is a closed economy with heterogeneous agents, rather than a small open economy, a key difference is that multiplicity arises in their model because of a price-dependent borrowing constraint, whereas the only market incompleteness in our setting is that outside investors cannot participate in markets

[2] Svensson (1981) also discusses price-contingent contracts in a temporary equilibrium model.

[3] More specifically, it relates to the literature on the importance of intermediary sector balance sheets such as Adrian and Boyarchenko (2012), He and Krishnamurthy (2013) and Krishnamurthy and Li (2020), among others.

[4] Seminal papers in the literature on fire sales includes Allen and Gale (1994) and Kiyotaki and Moore (1997); see Shleifer and Vishny (2011) for a broader survey.

[5] Importantly, even when we introduce state-contingent contracts, markets are still incomplete in the sense that outside investors do not participate in markets at date 0.

at date 0.

The interaction between multiple equilibria and the portfolio decisions of leveraged intermediaries in our paper is also similar to Bocola and Lorenzoni (2020)'s analysis of liability dollarization. In their open economy model, the fear of a currency crisis can be self-fulfilling because it leads consumers to demand dollar-denominated deposits, which in turn induces banks to borrow in dollars, leaving them exposed to crises. Similarly, in our environment, intermediaries' decision to issue safe assets can be self-fulfilling because households want to hedge the risk of a fall in the price of trees when the leverage decisions of intermediaries expose the economy to the risk of fire sales. Besides the difference in the focus of the two papers, the main difference between our analysis and theirs is that we focus on intermediaries' decision *whether* to issue safe assets, while they focus on intermediaries' choice between borrowing in domestic or foreign currency.

Our paper contributes to the recent literature studying the demand and supply of safe assets (Caballero, 2006; Caballero et al., 2017; Caballero and Farhi, 2018; Del Negro et al., 2017; Gorton and Ordoñez, 2020). The theoretical papers in this literature generally assume that investors face *fundamental risk*, e.g. an exogenous probability of a "disaster" (Barro et al., 2014; Caballero and Farhi, 2018) which causes them to demand safe assets as a hedge. In such a framework, a fall in the safe rate of interest can be attributed to an increase in fundamental risk (Acharya and Dogra, 2021). Our model instead suggests that an increased demand for safe assets, and a decline in the *natural rate of interest*, could arise endogenously as a result of private safe asset creation.[6]

Our paper also relates to the broader literature on sunspot equilibria. Cass and Shell (1983) showed that sunspots can affect real outcomes in economies in which there is limited participation in asset markets (e.g., because in an overlapping generations economy, participation is limited to those agents currently alive). Whereas Cass and Shell (1983) allow agents to trade assets with sunspot contingent payoffs, Mas-Colell (1992) and Gottardi and Kajii (1999) study economies in which asset payoffs do not depend on sunspot variables, i.e., markets are incomplete with respect to these sunspots. Relative to this literature, we are closest to Cass and Shell (1983) since asset markets in our economy are complete for all agents who can participate.[7] While this literature focuses on describing conditions un-

[6]Relatedly, Diamond (2020) shows how an exogenous increase in demand for safe assets can increase riskiness of financial intermediaries' portfolios in a model where financial intermediaries emerge endogenously. Segura and Villacorta (2020) shows that safe asset creation can increase the risk of originated loans. Infante and Ordoñez (2020) study how bias in the composition of collateral towards private assets relative to public assets interacts with aggregate volatility to determine insurance properties of private assets. Gottardi et al. (2021) study how assets generated from intermediation activity and pledged as collateral create fragility.

[7]Strictly speaking, since households in our model have Epstein-Zin preferences, it is closer to Balasko (1983) who extends the analysis of Cass and Shell (1983) beyond Von-Neumann Morgenstern preferences.

der which sunspot equilibria can exist, we describe how the introduction of new private safe assets can introduce sunspot equilibria which would not otherwise exist, and use this framework to study financial fragility.

Finally, our paper is also related to the macro-finance literature that attempts to explain the puzzlingly high volatility of asset prices and in particular the role of dynamic hedging strategies such as portfolio insurance. Grossman (1988) and Grossman and Zhou (1996) demonstrate that the demand for put options on a risky asset determines the asset's subsequent realized volatility.[8] Shin (2010) and Adrian and Shin (2013) argue that risk management tools such as Value at Risk can generate procyclical leverage and amplify the aggregate effect of shocks. We show that trading of insurance-like contracts gives rise to market volatility even in the absence of any fundamental risk. Gennotte and Leland (1990) and Jacklin et al. (1992) use frameworks with asymmetric information to study how portfolio insurance might have contributed to the October 1987 market crash. DeLong et al. (1990) show how rational speculation in the presence of positive feedback investment strategies may be destabilizing. Brunnermeier and Pedersen (2009) model sudden market illiquidity episodes in an environment with information asymmetries and fundamental volatility.[9] More recently, Malherbe (2014) shows self-fulfilling liquidity dry-ups may occur due to liquidity hoarding in markets featuring adverse selection mechanisms. In our setup, all market participants are symmetrically informed and there is no adverse selection problem.

The rest of the paper is organized as follows. Section 1 presents a baseline version of the economy without private safe assets. Section 2 introduces private safe assets in the form of insurance contracts, and shows that this gives rise to non-fundamental price volatility. Section 3 shows that the same non-fundamental price volatility also arises if intermediaries issue other kinds of safe assets. Section 4 describes how various policies can eliminate this non-fundamental volatility, and Section 5 concludes.

1 Baseline model: Economy without private safe asset creation

To study the interaction between private safe asset creation and risk created in the financial system, we begin by studying a version of our economy in which we deliberately shut down private safe asset creation. This will be a useful point of comparison when we introduce private safe asset creation in Section 2.

[8] Chowdhry and Nanda (1998) highlight that rigidity in trading rules (such as margin constraints) can lead to market instability when there are sequential trading opportunities.

[9] See also Brennan and Schwartz (1989), Basak (1995) and Oehmke (2014), among others.

Environment Time is discrete and lasts for two periods $t = 0, 1$. There are three sets of agents: (i) risk-averse households, (ii) risk-neutral financial intermediaries and (iii) outside investors. There are two goods: (i) cookies denoted by c and (ii) apples denoted by a. Cookies are perishable, and there is a fixed endowment of cookies at both dates 0 and 1. Each tree produces one apple at the end of date 1. Trees do not produce apples at date 0 but can be traded at date 0. There are no shocks to preferences, endowments or technology. Throughout this section we assume that the only assets traded are trees.

Households There is a unit mass of risk-averse households who only value the consumption of cookies. Households have infinite intertemporal elasticity of substitution but are risk-averse over date 1 consumption:

$$U^h(c_0, c_1) = c_0 + \left[\mathbb{E}c_1^{1-\gamma}\right]^{\frac{1}{1-\gamma}}, \gamma > 1$$

Households are born with an endowment of cookies χ_0^h at date 0 but have none at date 1. Furthermore, they are endowed with all the trees e in the economy (normalized to $e = 1$). The date 0 budget constraint of households can be written as

$$c_0^h + p_0 e^h = \chi_0^h + p_0 \tag{1}$$

where p_0 denotes the date 0 price of a tree and e^h denotes the measure of trees retained by households at date 0. The date 1 budget constraint can be written as

$$c_1^h = p_1 e^h \tag{2}$$

where p_1 is the (potentially stochastic) price of trees at date 1. Households choose c_0, c_1 and e^h to maximize their lifetime utility subject to the budget constraints (1)-(2) and non-negativity constraints on c_0, c_1 and e^h. The optimality condition of households is given by

$$p_0 = \frac{\mathbb{E}p_1 c_1^{-\gamma}}{\left[\mathbb{E}c_1^{1-\gamma}\right]^{\frac{\gamma}{\gamma-1}}} = \left[\mathbb{E}p_1^{1-\gamma}\right]^{\frac{1}{1-\gamma}} \tag{3}$$

where the second equality is from the period 1 budget constraint.[10]

[10]Throughout the paper we assume that χ_0^h is large enough that the non-negativity constraint on date 0 consumption does not bind. This condition is trivially satisfied in this section given that households cannot trade additional assets and $e^h \leq 1$.

Financial Intermediaries There is a unit mass of risk-neutral financial intermediaries (FIs) who are endowed with a *small* number of cookies $\chi_0^f < 1$ at date 0 and no cookies at date 1. χ_0^f can be interpreted as FI net worth, and the assumption that it is small implies that our economy features a relatively undercapitalized financial system; as is common in the literature on financial frictions, intermediary net worth must be sufficiently small for these frictions to have bite. FIs do not have any trees but regard apples and cookies as perfect substitutes. Their preferences are given by

$$U^f(c_0, c_1, a_1) = c_0 + \mathbb{E}(c_1 + a_1) \tag{4}$$

The date 0 budget constraint of FIs is given by

$$c_0^f + p_0 e^f = \chi_0^f \tag{5}$$

where e^f denotes the measure of trees that FIs buy from households. Their date 1 budget constraint is

$$c_1^f + p_1 a_1^f = p_1 e^f \tag{6}$$

FIs choose c_0^f, c_1^f, e^f and a_1^f to maximize their utility (4) subject to budget constraints (5)-(6) and non-negativity constraints on c_0^f, c_1^f, e^f and a_1^f. At date 1, FIs sell all their trees if $p_1 > 1$ and retain all their trees if $p_1 < 1$ (they are indifferent between holding and selling trees if $p_1 = 1$). At date 0, FIs spend all their endowment on trees if $p_0 < \mathbb{E}\max\{1, p_1\}$ and do not buy any trees if $p_0 > \mathbb{E}\max\{1, p_1\}$ (they are indifferent between buying and not buying trees if $p_0 = \mathbb{E}\max\{1, p_1\}$).

Outside Investors Finally, as in Stein (2012), there is a unit mass of outside investors who only trade and consume at date 1. They have a large endowment of cookies at date 1, χ_1. These are the only agents in the economy who have any cookies at date 1. The outside investors enjoy both cookies and apples; their preferences are given by

$$U^o(c_1^o, a_1^o) = v(a_1^o) + c_1^o$$

where $v(\cdot)$ is an increasing and concave function. The concavity of $v(\cdot)$ captures the idea that outside investors are non-specialists whose capacity to efficiently utilize trees features strongly diminishing returns (Shleifer and Vishny, 1992; Kiyotaki and Moore, 1997; Stein, 2012). They face the budget constraint

$$c_1^o + p_1 a_1^o = \chi_1.$$

The optimal demand for trees by outsiders is given by

$$\begin{aligned} v'(a_1^o) \leq p_1 \quad , \quad a_1^o \geq 0 \\ a_1^o \left[v'(a_1^o) - p_1\right] = 0 \end{aligned} \tag{7}$$

We assume $v'(0) > 1 > v'(1)$, which ensures that outsiders optimally purchase some interior quantity of trees $a_1^o \in (0, 1)$ when $p_1 = 1$.

1.1 Fundamental Equilibrium

An equilibrium is a collection of prices $\{p_0, p_1\}$ and quantities $\{c_0^h, c_1^h, e^h, c_0^f, c_1^f, a_1^f, e^f, c_1^o, a_1^o\}$ such that all agents optimize and markets for date 0 cookies, date 1 cookies, trees and date 1 apples clear:

$$c_0^h + c_0^f = \chi_0^h + \chi_0^f \tag{8}$$

$$c_1^h + c_1^f + c_1^o = \chi_1 \tag{9}$$

$$e^h + e^f = 1 \tag{10}$$

$$a_1^o + a_1^f = 1 \tag{11}$$

We characterize the equilibrium by backward induction. We start by characterizing the date 1 price of trees given the equilibrium tree-holdings of the household $e^h \in [0, 1]$.

Lemma 1 (Date 1 price of trees). *In equilibrium,* $p_1 = \min\{1, v'(e^h)\}$.

Proof. First, we show that at date 1, $p_1 \leq 1$. To see this, suppose that $p_1 > 1$. Then, FIs would sell all their trees, i.e., $a_1^f = 0$. Then in equilibrium, outside investors at date 1 must consume all apples and so $a_1^o = 1$. The optimality condition for outside investors (7) then requires that $v'(1) = p_1 > 1$, which contradicts our assumption that $v'(1) < 1$.

Next, consider the case where $p_1 < 1$. In this case, FIs prefer to retain all their trees at date 1, i.e., $a_1^f = 1 - e^h$. In equilibrium, outside investors must then purchase the remaining e^h trees. Their optimality condition (7) implies that $v'(e^h) = p_1$.[11] □

The date 1 price of trees is decreasing in e^h, the measure of trees retained by households at date 0 and sold to outside investors at date 1. Since outside investors have a downward

[11] In the case where $e^h = 0$, this condition becomes $v'(0) \leq p_1$.

sloping demand curve for trees (7), the more trees outsiders have to buy at date 1, the lower is the price p_1 they are willing to pay. Since FIs are always willing to sell trees for cookies at $p_1 = 1$, prices are bounded above by 1. If $e^h \leq \bar{e}$ where $\bar{e}$ is defined as the level of e^h for which $v'(\bar{e}) = 1$, FIs will sell their excess holdings of trees $\bar{e} - e^h$ to outside investors. If instead $e^h > \bar{e}$, households hold on to all their trees, and outsiders are the marginal buyers of trees. These two cases are depicted by the left and right segments of the dotted blue curve.

With that characterization of the date 1 price of trees, we can now characterize equilibrium at date 0: the price of trees and the measure of trees retained by households. First, note that since $p_1 \leq 1$, FIs will spend all their endowment of cookies on buying trees at date 0 if $p_0 < 1$, i.e., $p_0(1 - e^h) = \chi_0^f$. Thus, in equilibrium, we must have

$$p_0 = \min\left\{\frac{\chi_0^f}{1 - e^h}, 1\right\} \tag{12}$$

Equation (12) denotes the date-0 FI demand for trees as a function of p_0 and is shown by the dashed red curve in Figure 1. Optimal household behavior at date 0 is described by (3).

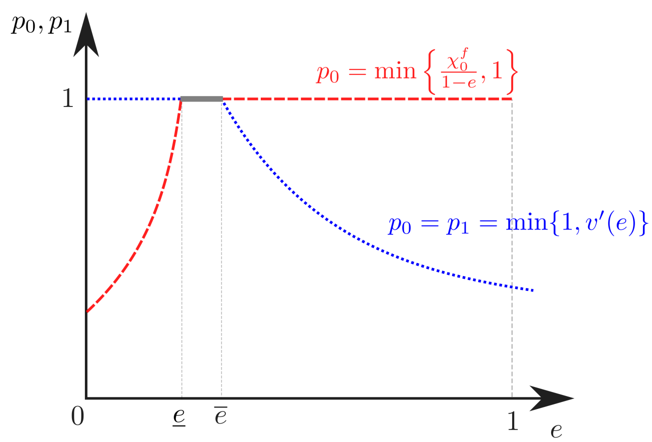

Figure 1: Equilibrium in the market for trees.

Given our characterization of date 1 prices, we can rewrite (3) as

$$p_0 = p_1 = \min\{1, v'(e^h)\} \tag{13}$$

which is denoted by the dotted blue curve in Figure 1. In equilibrium, if the quantity of trees retained by households is relatively high, households anticipate that there will be a glut of trees in the market at date 1, and their price will be relatively low. Anticipating this, they will only retain trees at date 0 if p_0 is sufficiently low. Finally, equilibrium is determined by the intersection of the dashed red and dotted blue curves, denoted by the solid gray line.

In all equilibria, trees have a price of 1 on both dates 0 and 1: $p_0 = p_1 = 1$ and outsiders consume $\bar{e}$ apples. There is one equilibrium in which households retain all the trees that will be ultimately sold to the outsiders (at date 1), selling only $1 - \bar{e}$ to the FIs (at date 0) who simply consume the apples at date 1. There is another equilibrium in which FIs spend all their resources to buy $1 - \underline{e}$ trees at date 0 and then sell $\bar{e} - \underline{e}$ to the outside investors at date 1. There is naturally a continuum of intermediate cases in between these two extremes. The following Proposition summarizes this result.

Proposition 1 (Fundamental Equilibrium)**.** *In equilibrium, the date 0 and date 1 price of trees is given by $p_0 = p_1 = 1$ and the measure of trees retained by households e^h lies in a closed interval $[\underline{e}, \bar{e}]$ where $\underline{e} = 1 - \chi_0^f$ and $\bar{e}$ is implicitly defined by $v'(\bar{e}) = 1$.*

Welfare Since $p_0 = p_1 = 1$ in all fundamental equilibria described in Proposition 1 above, it is straightforward to see that the welfare of households, FIs and outside investors is given by

$$U^h = \chi_0^h + 1 \qquad U^f = \chi_0^f \qquad U^o = v(\bar{e}) - \bar{e} \tag{14}$$

2 Endogenous fragility

In the environment just described, trees were the only assets households could use to transfer wealth from date 0 to date 1. In equilibrium, households carry some fraction of trees into date 1 and sell them directly to the outside investors. Even though households are risk averse, they are happy to retain a fraction of trees as they are riskless assets – their price never falls below 1. Given this, even if households had the option to purchase insurance against a fall in prices, they would not buy such insurance at any positive price since the event has zero probability. Suppose, however, that for some reason there is a positive probability that the price of trees at date 1 is less than 1. This would expose households to the risk of low consumption if the price falls at date 1; households would like to insure against this risk, if possible.

We now allow FIs to sell such insurance but keep the rest of the environment as in Section 1. As we will show, allowing FIs to trade insurance can give rise to endogenous financial fragility, in which asset prices fall in some states of the world even absent fundamental shocks. More specifically, we consider a security which is designed to insure households against the price of trees: it pays out $1 - p_1$ cookies if the realized price of trees $p_1 < 1$. This can be thought of as a put option with a strike price of 1. In order to show that financial fragility can arise endogenously in equilibrium, we start by describing how the problem of each agent changes with the introduction of this additional financial instrument.

Households With the introduction of insurance, households' budget constraints now become

$$\begin{aligned} c_0^h + p_0 e^h + q z^h &= \chi_0^h + p_0 && (15) \\ c_1^h &= p_1 e^h + (1 - p_1) z^h && (16) \end{aligned}$$

where q denotes the date 0 price of a unit of insurance and z^h denotes the quantity of insurance purchased by households. Plugging the budget constraints into the households' objective function yields

$$\max_{e^h, b^h} \chi_0^h + p_0 - p_0 e^h - q z^h + \mathbb{E}\left[\left(p_1 e^h + (1 - p_1) z^h\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$$

where the expectation is over the realization of the price of trees at date 1.

Financial Intermediaries Similarly, the date 0 and date 1 budget constraints of FIs can be written as

$$\begin{aligned} c_0^f + p_0 e^f &= \chi_0^f + q z^f && (17) \\ c_1^f + p_1 a_1^f + (1 - p_1) z^f &= p_1 e^f && (18) \end{aligned}$$

Given that FIs' consumption cannot be negative in any state of the world, their issuance of insurance is constrained by the amount that they can pay out at date 1 in the state where a low price is realized, i.e.,

$$(1 - p_1)\, z^f = p_1 \left(e^f - a_1^f\right) - c_1^f \leq p_1 e^f \tag{19}$$

in all states of the world, where z^f denotes the amount of insurance issued by FIs at date 0. (19) states that in any state of the world, FIs must be able to pay out on their insurance contract by raising funds through the sale of trees, i.e., the sale of insurance must be backed by holdings of trees.[12] But the purchase of trees at date 0 can be partially financed by the sale of insurance, allowing FIs to purchase more trees (given prices) than would be possible absent insurance.

[12]Note that FIs cannot default on their obligations; in this sense, our notion of "risky" and "safe" assets captures liquidation risk rather than default risk.

Outside Investors Since outside investors do not participate in markets at date 0, they cannot buy or sell insurance, and their budget constraint is unchanged.

2.1 Equilibrium with Insurance

Equilibrium is defined as a collection of prices $\{p_0, p_1, q\}$ and quantities $\{c_0^h, c_1^h, e^h, z^h, c_0^f, c_1^f, a_1^f, e^f, z^f, c_1^o, a_1^o\}$ such that all agents optimize and markets clear. In addition to the market clearing conditions (8)-(11), the market for insurance must clear

$$z^h = z^f \tag{20}$$

It is straightforward to see that the fundamental equilibrium described in Section 1 continues to be an equilibrium where the price of insurance $q = 0$ and the date 1 price of trees is degenerate and equal to 1:

Lemma 2. *The fundamental equilibrium described in Section 1 continues to be an equilibrium where the price of insurance $q = 0$ and the date 1 price of trees is degenerate and equal to 1.*

Proof. Suppose $q = 0$ and $z^f = z^h = 0$ and all other prices and allocations are as described in Proposition 1. Given that p_1 always equals 1, insurance contracts always pay out $1 - p_1 = 0$. Clearly, the possibility of trading assets with a price of 0 and payoff of 0 does not change any agent's optimal decisions. Thus, the fundamental equilibrium satisfies all equilibrium conditions in the economy with insurance. $\square$

However, there also exist equilibria in which insurance trades at a positive price and the date 1 price of trees is stochastic. In these equilibria, households demand insurance because they seek to hedge against price volatility. But this price volatility arises only because the sale of this insurance leaves intermediaries exposed to fire sales which cause price fluctuations.

More specifically, suppose that agents in the economy expect that with some probability $\lambda > 0$, the price of trees p_1 falls below its fundamental value of 1 and equals $\underline{p} = v'(1) < 1$, while with probability $1 - \lambda$, the price of trees is equal to its fundamental value $p_1 = 1$. $\lambda \in (0, 1)$ is the probability of a sunspot which results in a low realization of the price of trees at date 1. Further suppose that the issuance constraint on FIs (19) binds when $p_1 = \underline{p}$

$$\left(1 - \underline{p}\right) z^f = \underline{p} e^f \qquad \Rightarrow \qquad \frac{z^f}{e^f} = \frac{\underline{p}}{1 - \underline{p}} \equiv \phi \tag{21}$$

We will show that such expectations can be sustained in equilibrium. First we show that such a putative equilibrium satisfies all date 1 equilibrium conditions. When $p_1 = \underline{p}$, FIs must sell all their trees to pay out on the insurance contracts they issued. Thus, in equilibrium, all trees must be purchased by outside investors who are only willing to pay a price of $v'(1) = \underline{p} < 1$ – confirming the low price. Conversely, if $p_1 = 1$, FIs need not sell any trees to outside investors, sustaining $p_1 = 1$ as in the fundamental equilibrium.

Optimal Behavior of FIs Given the equilibrium at date 1, next we describe the behavior of agents at date 0. Given (21), the date 0 problem of FIs can be written as

$$\max_{e^f, z^f} \chi_0^f - p_0 e^f + q z^f + \mathbb{E}\left[e^f - \frac{1-p_1}{p_1} z^f\right]$$

s.t.

$$\begin{aligned} \chi_0^f - p_0 e^f + q z^f &\geq 0 &(22)\\ \left(1-\underline{p}\right) z^f &\leq \underline{p} e^f &(23) \end{aligned}$$

where the objective function uses the fact that $p_1 \leq 1$, and so any remaining cookies that the FIs have after insurance payouts are used to buy trees and eat the apples they produce. The first constraint (22) is the non-negativity constraint on date 0 consumption, indicated by area above the dashed red line in Figure 2. The second constraint imposes that FIs must be able to pay out insurance claims even after the lowest realization of $p_1 = \underline{p}$, given that date 1 consumption must be non-negative. This constraint can be written as $z^f \leq \phi e^f$ where $\phi = \frac{\underline{p}}{1-\underline{p}}$ and is denoted by the area below the solid blue line. FIs' indifference curves are depicted by the dotted purple lines.[13] Overall, the feasible set of choices that FIs can make is denoted by the shaded gray triangle.

When both constraints bind, the quantity of trees purchased by FIs is

$$e^f = \frac{\chi_0^f}{p_0 - \phi q} \geq \frac{\chi_0^f}{p_0} \tag{24}$$

That is, the maximum quantity of trees that an FI can buy (the intersection of the solid blue and dashed red lines) is greater than χ_0^f / p_0, the quantity they could purchase using all of their endowment (the intersection of the dashed red line and the horizontal axis).

Buying trees increases the amount of insurance that can be issued, issuing insurance provides more funds with which to purchase trees, and so forth.[14]

[13] Strictly speaking, these are not indifference curves since we have already plugged in the expression for c_0^f. As a result, the slope of these curves depends on the asset prices.

[14] If insurance were sufficiently expensive ($q \geq p_0/\phi$), this would be so profitable that the FIs problem would have no solution (the solid blue line would be steeper than the dashed red line, and the grey area

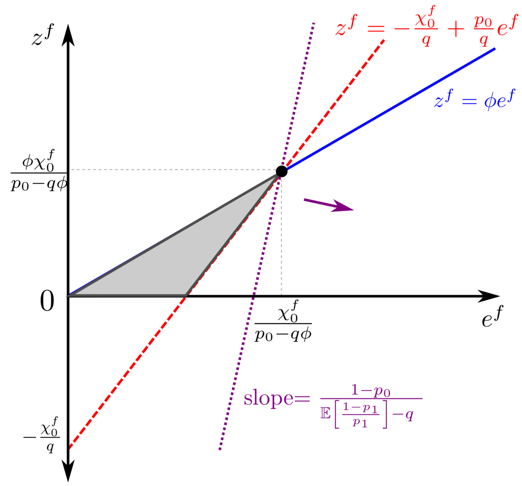

Figure 2: Date 0 problem for FI's

While FIs always *can* purchase $\frac{\chi_0^f}{p_0 - \phi q}$ trees, whether they want to do so depends on their induced preferences over insurance and trees. Graphically, it is clear that they will purchase the maximum quantity of trees if their indifference curves are steeper than $\frac{p_0}{q}$, i.e., if

$$\frac{1}{p_0} > \frac{1}{q} \mathbb{E}\left[\frac{1 - p_1}{p_1}\right] \tag{25}$$

The LHS and RHS of (25) can be interpreted as the gross expected return on trees and insurance respectively, weighted by the marginal value of cookies to FIs at date 1. A tree at date 0 guarantees the FI one apple at date 1 which has a marginal utility of 1. Equivalently, the tree can be sold for p_1 cookies, and the FIs marginal value of cookies is $1/p_1$, making the marginal value weighted expected return $\frac{\mathbb{E}[p_1/p_1]}{p_0} = 1/p_0$. Conversely, one unit of insurance pays out $1 - p_1$ cookies at date 1 and yields a marginal value weighted expected return $\frac{1}{q}\mathbb{E}\left[\frac{1-p_1}{p_1}\right]$. (25) states that trees must yield a higher expected return than insurance if FIs are to sell insurance and buy trees. We conjecture that in equilibrium this condition is satisfied; we will show that, if $\lambda > 0$ is not too large, this is indeed the case.

When (25) is satisfied, FIs purchase $\frac{\chi_0^f}{p_0 - \phi q}$ trees and sell $\frac{\phi_0 \chi_0^f}{p_0 - \phi q}$ insurance claims backed by the trees. This yields a payoff of $\frac{\chi_0^f}{p_0 - \phi q}$ if $p_1 = 1$ and 0 if $p_1 = \underline{p} < 1$. Intuitively, it is as if FIs make a leveraged bet that p_1 will equal 1, using their whole endowment to purchase an "Arrow security" which pays out one cookie if $p_1 = 1$ and nothing otherwise. This security has a price $p_0 - \phi q$; it is constructed synthetically by taking a long position in a tree (which costs p_0 cookies) and selling the payoffs from the tree which occur when $p_1 = \underline{p} < 1$ (which raises ϕq cookies).

would be unbounded). This can never happen in equilibrium.

Optimal Behavior of Households Next we describe optimal household behavior at date 0. Here we can use the fact that households consume $\underline{p}$ cookies after a low realization of p_1. Their optimization problem yields the following Euler equations for trees and insurance respectively

$$p_0 = \frac{\lambda \underline{p}^{1-\gamma} + (1-\lambda)(e^h)^{-\gamma}}{\left[\lambda \underline{p}^{1-\gamma} + (1-\lambda)(e^h)^{1-\gamma}\right]^{-\frac{\gamma}{1-\gamma}}} \tag{26}$$

$$q = \frac{\lambda(1-\underline{p})\underline{p}^{-\gamma}}{\left[\lambda \underline{p}^{1-\gamma} + (1-\lambda)(e^h)^{1-\gamma}\right]^{-\frac{\gamma}{1-\gamma}}} \tag{27}$$

Combining (26)-(27) we get

$$q^a \equiv p_0 - \frac{\underline{p}}{1-\underline{p}} q = \frac{(1-\lambda)(e^h)^{-\gamma}}{\left[\lambda \underline{p}^{1-\gamma} + (1-\lambda)(e^h)^{1-\gamma}\right]^{-\frac{\gamma}{1-\gamma}}} \tag{28}$$

Intuitively, this equation prices the Arrow security described earlier which pays out 1 cookie when $p_1 = 1$.

Combining (28) with the FIs demand for these claims (24) and using the fact that $e^h = 1 - e^f$ we have

$$\frac{(1-\lambda)(e^h)^{-\gamma}(1-e^h)}{\left[\lambda \underline{p}^{1-\gamma} + (1-\lambda)(e^h)^{1-\gamma}\right]^{-\frac{\gamma}{1-\gamma}}} = \chi_0^f \tag{29}$$

The LHS of the expression above is strictly decreasing, so this defines a unique solution for e^h. It remains to check that the inequality (25) holds, i.e., it is indeed optimal for FIs to lever up to the maximum. Using equations (26)-(27), the inequality (25) simplifies to

$$e^h > \underline{p}^{\frac{\gamma-1}{\gamma}} \tag{30}$$

Since e^h is decreasing in χ_0^f, this inequality is satisfied as long as χ_0^f is small. Thus, under this assumption there do indeed exist equilibria in which insurance creates endogenous price volatility, as Proposition 2 summarizes:

Proposition 2 (Insurance Equilibrium). *If $\chi_0^f < 1 - \underline{p}^{\frac{\gamma-1}{\gamma}}$, for every $\lambda \in \left(0, \overline{\lambda}\right)$ where $\overline{\lambda} < 1$ is implicitly defined by*

$$\chi_0^f = \frac{\left(1-\overline{\lambda}\right)\left[1 - \underline{p}^{\frac{\gamma-1}{\gamma}}\right]}{\left[\overline{\lambda}\underline{p}^{\frac{1-\gamma}{\gamma}} + 1 - \overline{\lambda}\right]^{\frac{\gamma}{\gamma-1}}} \tag{31}$$

there exists an equilibrium in which $p_1 = 1$ *with probability* $1 - \lambda$ *and* $p_1 = \underline{p} = v'(1) < 1$ *with probability* λ. e^h *is implicitly defined by equation* (29). p_0 *and* q *are defined by* (26) *and* (27) *and* $z_h = \frac{\underline{p}}{1-\underline{p}}(1 - e^h)$.

Importantly, our model provides not only a theory of self-fulfilling price declines, but also a theory of self-fulfilling leverage build-ups. Within an equilibrium with $0 < \lambda < \overline{\lambda}$, price declines at date 1 can be self-fulfilling: when p_1 is low, FIs sell trees to pay out on their insurance contracts, pushing down the price, while when $p_1 = 1$, they do not need to do so and there is no downward pressure on prices. But in addition, a high (or low) *probability* of price declines can be self-fulfilling. If households anticipate that prices might fall, they demand insurance from FIs, whose issuance of insurance actually makes price declines possible. Conversely, if households did not believe that price declines were possible, they would not demand insurance, FIs would not take on leverage and price declines would never occur. To look at this another way, the supply of private safe assets may create its own demand – by providing insurance against adverse financial outcomes, FIs become vulnerable to fire sales which ironically increase households' need for insurance.

Cass and Shell (1983) showed that sunspots cannot affect the allocation of resources in complete market economies in which agents share the same probability beliefs. Proposition 2 does not contradict this result as our economy features market incompleteness: outside investors cannot trade securities at date 0. In fact, Appendix F shows that if we allowed outside investors to participate in markets at date 0, only the fundamental equilibrium defined in Proposition 1 would exist.

2.2 Welfare

The non-fundamental equilibria just described feature more volatile prices than the fundamental equilibrium with the amount of this volatility increasing in λ. Perhaps unsurprisingly, households are worse off in any non-fundamental equilibrium with insurance than in the fundamental equilibrium. To see this, first we need to characterize how households' consumption after a high price realization, i.e., their holding of trees, depends on λ. Equation (29) implicitly describes e^h as a decreasing function of λ. Intuitively, if λ is higher, there is greater risk that the price of trees will fall and households are less willing to remain exposed to this risk.

Lemma 3. *In equilibrium,* e^h *is strictly decreasing in* λ *(the probability of low price realization) with* $e^h \to \underline{e}$ *as* $\lambda \to 0$. *Furthermore,* e^h *is increasing in* $\underline{p}$ *(the low price realization) and decreasing in* χ_0^f *(the net worth of FIs)*

Proof. The proof is an application of the implicit function theorem on (29) and using the fact that in equilibrium $e^h > \underline{p}$. $\square$

Households' utility in an equilibrium with insurance in which $p_1 = \underline{p}$ with probability λ is

$$U^h_{\text{insurance}} = \underbrace{\chi^f_0 + \chi^h_0}_{c^h_0} + \left[\lambda \underline{p}^{1-\gamma} + (1-\lambda)\left(e^h(\lambda)\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$$

As $\lambda \to 0$, the welfare of households converges to the welfare in the fundamental equilibrium (14).[15] Clearly, higher λ reduces household welfare since $\underline{p} < e^h(\lambda)$: households are less than fully insured and consume less when prices fall. In addition, higher λ indirectly reduces welfare by reducing households' tree holdings (e^h is decreasing in λ, c.f. Lemma 3) and hence their consumption when the price does not fall. Thus, household welfare is unambiguously decreasing in λ – implying that household welfare is always lower in an equilibrium with insurance relative to the fundamental equilibrium.

In contrast, FIs are weakly better off in any equilibrium with insurance than in the fundamental equilibrium. Recall that in the fundamental equilibrium, FIs welfare is χ^f_0 – the same utility they would obtain if they consumed their endowment and did not trade. Since FIs still have the option of consuming their endowment and refraining from trade, the welfare accruing to them from following their optimal strategy must be at least as large as χ^f_0.[16]

Like the FIs, outside investors are better off in any equilibrium with insurance. With probability $1-\lambda$, they receive the same utility as in the fundamental equilibrium, and with probability λ, they have the opportunity to buy apples at fire-sale prices yielding strictly higher utility

$$U^o_{\text{insurance}} = (1-\lambda)\left[v(\bar{e}) - \underbrace{v'(\bar{e})}_{=1}\bar{e}\right] + \lambda\left[v(1) - v'(1)\right] > v(\bar{e}) - \bar{e},$$

where the inequality follows from the concavity of $v()$.

Proposition 3 (Welfare gains)**.** *Households are worse off, financial intermediaries are weakly better off, and outside investors are better off in any equilibrium in which insurance trades at a positive price relative to their welfare levels in the fundamental equilibrium.*

[15] As $\lambda \to 0$, we have $U^h_{\text{insurance}} \to \chi^f_0 + \chi^h_0 + e^h$. Also, with $\lambda \to 0$, (29) implies that $e^h = 1 - \chi^h_0$. Combining these yields $U^h_{\text{insurance}} \to \chi^f_0 + 1$ which is the same as in the fundamental equilibrium (14).

[16] Formally, note that the FI's welfare is $(1-\lambda)(1-e^h) = (1-\lambda)\frac{\chi^f_0}{p_0 - \phi q} = \chi^f_0 \frac{\left[\lambda \underline{p}^{1-\gamma} + (1-\lambda)(e^h)^{1-\gamma}\right]^{-\frac{\gamma}{1-\gamma}}}{(e^h)^{-\gamma}}$. From inequality (30), we know that $e^h > \underline{p}$). Then, it follows that this welfare in the insurance equilibrium is greater than χ^f_0, which is their welfare in the fundamental equilibrium.

3 Financial fragility with other private safe assets

The way in which private safe asset creation enabled non-fundamental price volatility to emerge in the economy just described is not restricted to the particular form of safe asset that we allowed FIs to issue in that economy, namely insurance contracts or put options. The same outcomes can arise if we allowed FIs to instead issue risk-free non-state-contingent bonds which pay one cookie to the holder at date 1. As we will explain in more detail, since these bonds are backed by FIs' holdings of trees, they can be interpreted as a repo transaction.

Compared to the environment described in Section 1, households' budget constraints now become

$$\begin{aligned} c_0^h + p_0 e^h + q^b b^h &= \chi_0^h + p_0 && (32)\\ c_1^h &= p_1 e^h + b^h, && (33) \end{aligned}$$

where q^b denotes the date 0 price of a bond and b^h denotes the quantity of bonds purchased by households. Similarly, the date 0 and date 1 budget constraints of FIs can be written as

$$\begin{aligned} c_0^f + p_0 e^f &= \chi_0^f + q^b b^f && (34)\\ c_1^f + p_1 a_1^f + b^f &= p_1 e^f && (35) \end{aligned}$$

Given that FIs' consumption cannot be negative in any state of the world, their debt issuance is constrained by the amount that they can pay out at date 1, i.e.,

$$b^f = p_1 \left(e^f - a_1^f\right) - c_1^f \leq p_1 e^f \tag{36}$$

in all states of the world, i.e., FIs must be able to pay out their debt obligations by raising funds through the sale of trees. Finally, since outside investors do not participate in markets at date 0, their budget constraint is unchanged.

Equilibrium Equilibrium is defined as a collection of prices $\{p_0, p_1, q\}$ and quantities $\{c_0^h, c_1^h, e^h, b^h, c_0^f, c_1^f,\ a_1^f, e^f, b^f, c_1^o, a_1^o\}$ such that all agents optimize and markets clear. In addition to the market clearing conditions (8)-(11), the market for bonds must clear: $b^h = b^f$.

As in the economy with insurance, the economy with bonds can experience non-fundamental asset price fluctuations. In fact, the set of equilibria is identical in the two economies – for every equilibrium in the economy with insurance, there exists a corresponding equilibrium in the economy with bonds with the same consumption profile across all agents and the same price of trees at all dates and states.

Proposition 4 (Non-fundamental equilibrium with bonds)**.** *For every equilibrium in the economy with insurance, there exists a corresponding equilibrium in the economy with bonds with the same values of consumption across all agents* $\{c_0^h, c_1^h, c_0^f, c_1^f, a_1^f, c_1^o, a_1^o\}$ *and prices* $\{p_0, p_1\}$*. Thus, if* $\chi_0^f < 1 - \underline{p}^{\frac{\gamma-1}{\gamma}}$*, and* $\bar{e} > \underline{p}^{-\left(\frac{1-\gamma}{\gamma}\right)}$*, then for every* $\lambda \in \left(0, \overline{\lambda}\right)$ *where* $\overline{\lambda} \in (0,1)$ *is implicitly defined by* (31)*, there exists an equilibrium in which* $p_1 = 1$ *with probability* $1 - \lambda$ *and* $p_1 = \underline{p} = v'(1) < 1$ *with probability* λ*.* e^h *is implicitly defined by*

$$\frac{(1-\lambda)\left[\left(1-\underline{p}\right)e^h + \underline{p}\right]^{-\gamma}\left(1-\underline{p}\right)\left(1-e^h\right)}{\left\{\lambda \underline{p}^{1-\gamma} + (1-\lambda)\left[\left(1-\underline{p}\right)e^h + \underline{p}\right]^{1-\gamma}\right\}^{-\frac{\gamma}{1-\gamma}}} = \chi_0^f \tag{37}$$

while the price of a tree p_0 *and the bond price is given by*

$$p_0 = \frac{\lambda\left(\underline{p}\right)^{1-\gamma} + (1-\lambda)\left[\left(1-\underline{p}\right)e^h + \underline{p}\right]^{-\gamma}}{\left\{\lambda \underline{p}^{1-\gamma} + (1-\lambda)\left[\left(1-\underline{p}\right)e^h + \underline{p}\right]^{1-\gamma}\right\}^{-\frac{\gamma}{1-\gamma}}} \tag{38}$$

$$q^b = \frac{\lambda\left(\underline{p}\right)^{-\gamma} + (1-\lambda)\left[\left(1-\underline{p}\right)e^h + \underline{p}\right]^{-\gamma}}{\left\{\lambda \underline{p}^{1-\gamma} + (1-\lambda)\left[\left(1-\underline{p}\right)e^h + \underline{p}\right]^{1-\gamma}\right\}^{-\frac{\gamma}{1-\gamma}}} \tag{39}$$

Proof. See Appendix B. □

As in the economy with insurance, self-fulfilling non-fundamental price volatility can arise due to the interaction between households' desire to hedge against price declines and FIs' leverage decisions which expose them to fire sales. However, this interaction now takes a different form. Rather than buying insurance to hedge the risk that the price of trees will fall, households sell a greater fraction of their endowment of trees and buy risk-free bonds which pay off in all states of the world. Likewise, while in the insurance economy, FIs only need to sell trees following a fall in price which obliges them to pay out on their insurance contract, here intermediaries sell trees in all states of the world to pay the bond holders. FIs are forced to sell more trees to meet their liability when the price of trees is lower, which in turn reinforces the lower price. By contrast, in the fundamental equilibrium, households are indifferent between holding trees and bonds since both are riskless assets, the spread between the expected return of these assets is zero, and FIs have no incentive to lever up to a point where they become vulnerable to fire sales.

Since the bonds issued by the intermediaries are backed by holdings of trees and are repaid in all states of the world, they can be interpreted as repo contracts. Under this interpretation, at date 0, households buy e^f trees from intermediaries, which have market value p_0e^f, and pay $q^b\underline{p}e^f$ cookies for them.[17] The FIs promise to repurchase these trees at price $\underline{p}$, i.e., they will repay $\underline{p}e^f$ cookies to households at date 1. The implicit haircut is $1 - q^b\frac{\underline{p}}{p_0} = \frac{\chi_0^f}{e^f}$. If we interpret bonds as repo contracts, the interplay between leverage and price volatility in the model is analogous to the relationship between the repo, US treasury and MBS markets in 2020. As the treasury market came under stress in March, hedge funds with a long position in cash treasuries were forced to partially unwind these positions, reinforcing the price declines through a feedback loop. Similarly, mortgage REITs with leveraged positions in agency MBS were forced to unwind these positions when MBS prices declined, creating another feedback loop. While this episode clearly coincided with a fundamental shock (COVID-19), the price fluctuations during this episode were widely perceived to be too large to be wholly attributable to a change in fundamentals (Cheng et al., 2020). Our model suggests that such volatility can arise endogenously, even absent fundamental shocks.

The trade between FIs and households can also be interpreted as a total return swap. The FIs are analogous to hedge funds who enter into a contract with investment banks (households) in which the FIs receive the return on the underlying asset (the tree) and make payments based on a pre-set rate (the interest rate on the risk-free bond). Under this interpretation, the households buy e^f trees on behalf of FIs; FIs put up an initial margin of χ_0^f and borrow the remaining amount $q^bb^f = p_0e^f - \chi_0^f$. At date 1, households pay FIs the gross return on the reference asset p_1e^f, net of the preset rate $\frac{p_0e^f - \chi_0^f}{q^b}$. If $p_1 = 1$, this net return is positive and FIs use the payment they receive from households to buy trees, supporting a high price. If $p_1 = \underline{p}$, the net payment is zero and households sell all their holdings of trees to outside investors. One interpretation of this sale is that FIs face a margin call (since the value of their position has fallen to zero) which they cannot meet, leading the households to liquidate the assets held in their account. This forced selling reinforces the low price. This self-fulfilling dynamic leading to fire sales is similar to the experience of Archegos Capital Management in early 2021. Archegos had entered into total return swap contracts with a number of large banks taking on positions concentrated in a small number of blue-chip companies. Archegos failed to meet its margin call when the price of the underlying shares declined, and its counter-parties unwound their position, reinforcing the price decline.

In addition, our model provides a novel perspective on the determinants of the safe rate of interest. A large recent literature has explored how safe assets such as US treasuries trade

[17] Of course, since FIs are not initially endowed with trees, they need to purchase these e^f trees from households at price p_0 before they can resell them as part of the repurchase agreement.

at a lower yield due to their safety and liquidity properties (Krishnamurthy and Vissing-Jorgensen, 2012). The theoretical literature on this topic generally assumes that the risk faced by investors which leads them to demand safe assets is fundamental, e.g. there is some exogenous probability of a "disaster" in which GDP falls substantially (Barro et al., 2014; Caballero and Farhi, 2018). According to this view, an decline in the safe rate of interest together with an increase in the spread between risky and safe assets, as observed over the past two decades, could be explained by an increase in demand for safe assets caused by an exogenous increase in fundamental risk (Acharya and Dogra, 2021). Our model instead suggests that the safe rate of interest could fall endogenously as a *result* of excessive private safe asset creation. An increase in borrowing by financial intermediaries leaves them vulnerable to fire-sales, increasing the risk that the assets held by households might fall in value and increasing their demand for safe assets. The following lemma shows formally that non-fundamental equilibria are associated with a lower safe rate of interest, i.e., a higher price of bonds.

Lemma 4. *In the economy with bonds, the price of bonds is higher in any non-fundamental equilibria than in the fundamental equilibrium.*

Proof. The Euler equation for the risk-free bonds is

$$q^b = \frac{\lambda \underline{p}^{-\gamma} + (1-\lambda)\left(e^h\right)^{-\gamma}}{\left[\lambda \underline{p}^{1-\gamma} + (1-\lambda)\left(e^h\right)^{1-\gamma}\right]^{-\frac{\gamma}{1-\gamma}}}$$

In the fundamental equilibrium with $\lambda = 0$, $q^b = 1$. In any non-fundamental equilibrium ($\lambda > 0$), we have

$$q^b = \frac{\mathbb{E}\left(c_1^h\right)^{-\gamma}}{f^{-1}\left[\mathbb{E} f\left(\left(c_1^h\right)^{-\gamma}\right)\right]} > 1$$

where $f(c) = c^{1-\gamma^{-1}}$ is concave and so the inequality follows from Jensen's inequality. $\square$

4 How policy can eliminate financial fragility

The reason that self-fulfilling falls in prices can occur in our environment is that FIs, who should be the "natural" buyers of trees at date 1, take on so much leverage that they are forced to *sell* these assets in some states of the world. Thus, a simple (albeit extreme) way for a policymaker to prevent such price volatility is to forbid intermediaries from issuing insurance, bonds or other private safe assets. This would return us to the economy without insurance (or bonds) described in Section 1 in which only the fundamental equilibrium exists.

Clearly, a strict enough tax or leverage restriction would have the same effect. However, even without directly forbidding or taxing private safe asset creation, policymakers can discourage it, and prevent financial fragility, by reducing the excess returns to leveraged investments in risky assets. Indeed, a wide variety of crisis-fighting policies can be interpreted as reducing this excess return. Loosely speaking, we can divide these policies into those which crowd out private safe asset creation by increasing the supply of "publicly backed" safe assets, and those which reduce the demand for private safe assets.

4.1 Increasing the supply of publicly backed safe assets

Public safe asset creation Policymakers can directly increase the supply of safe assets by issuing public safe assets themselves (e.g., government bonds or reserves held at the central bank). We now introduce a government into the economy with bonds which can issue risk-free bonds and buy trees at date 0 and can levy lump sum taxes T on outside investors at date 1. The date 0 and date 1 budget constraint of the government can be written as

$$q^b b^g = p_0 e^g \tag{40}$$

$$T + p_1 e^g = b^g \tag{41}$$

where b^g denotes the face value of government debt issued and e^g denotes the government's holdings of trees. The budget constraints of all other agents are unchanged except that the budget constraint of outside investors becomes

$$c_1^o + p_1 a_1^o = \chi_1 - T$$

Finally, market clearing conditions for trees is given by

$$e^h + e^g + e^f = 1 \tag{42}$$

while the bond market clearing requires $b^f + b^g = b^h$.

Issuing public debt does not change consumption allocations in the fundamental equilibrium. This is because both debt and trees are safe assets and trade at a price of 1, and the government never needs to raise taxes on outside investors at date 1. The only difference is that some households or FIs who would have purchased trees at date 0 instead purchase government debt.

However, in a non-fundamental equilibrium, trees are risky assets, and so an increase in the supply of public safe assets does affect consumption allocations across agents. Household consumption when $p_1 = \underline{p}$ is now equal to $\underline{p} + \underline{T}$ where $\underline{T}$ denotes taxes raised on outside investors by the government when $p_1 = \underline{p}$. Higher b^g raises T and raises household consumption after a fall in tree prices, reducing the difference in household consumption between the two states at date 1. This in turn reduces risk premia, reducing the price of bonds and raising the date 0 price of trees, and thus, reducing FIs incentive to take a levered position in trees. When b^g is high enough, the risk premium is so low that FIs strictly prefer not to take a leveraged position in trees, eliminating the non-fundamental equilibrium. Public safe asset creation crowds out socially risky private safe asset creation.

Proposition 5 (Public provision of safe assets)**.** *If the government issues* $b^g \geq b^* \equiv \frac{\underline{p}^{\frac{1}{\gamma}}}{1-\underline{p}}\left(1 - \chi_0^f\right) - \frac{\underline{p}}{1-\underline{p}}$ *debt, no non-fundamental equilibrium exists.*

Proof. See Appendix C. □

Interestingly enough, while the government's fiscal capacity is essential in allowing it to rule out the non-fundamental equilibrium, it is not actually necessary to raise taxes on equilibrium. This is because in the fundamental equilibrium, the debt can be repaid in full by selling the government's holding of trees to FIs. It is also important to note that the government's ability to raise taxes does not preclude it from having to sell trees at fire sale prices if $p_1 = \underline{p}$. Indeed, our maintained assumption is that the government takes p_1 as given and always sells its entire holdings of trees at date 1, raising taxes only to repay the remaining debt. Instead, fiscal capacity is valuable because it allows the government to increase the total supply of safe assets, satiating the households' demand for safety and reducing FIs desire to take levered positions. Absent government debt, private safe asset creation can never raise household consumption following a fall in prices above $\underline{p}$, the proceeds from selling all the trees in the economy to outside investors. Public safe asset creation *can* do this, raising household consumption to $\underline{p} + \underline{T}$, because the government can also tax outside investors.

While we have described the safe asset b^g as a government bond, it can also be interpreted as the liability of the central bank, e.g., interest-bearing reserves or reverse repos. In this light, our finding that increased public safe assets can improve financial stability resonates with Greenwood et al. (2016)'s argument that

> the Fed can complement its regulatory efforts on the financial-stability front by maintaining a relatively large balance sheet, even when policy rates have moved well away from the zero lower bound (ZLB). In so doing, it can help ensure that there is an ample supply of government-provided safe short-term claims—e.g., interest-bearing reserves and reverse repurchase agreements. By expanding the

overall supply of safe short-term claims, the Fed can weaken the market-based incentives for private- sector intermediaries to issue too many of their own short-term liabilities.

Consistent with their argument, our model suggests that the key to discouraging excessive private risk taking is to reduce the premium on money like claims. In our stylized two-asset model, this is simply the spread between trees and bonds; sufficiently reducing this spread eliminates FIs' incentive to take on maximum leverage and rules out non-fundamental equilibria.

Transfers to financial intermediaries Rather than issuing debt ex-ante, the government could simply use its fiscal capacity to make transfers to intermediaries in a crisis. These transfers can be interpreted as bailouts of financial institutions as observed in 2008 and after other financial crises. A common concern regarding bailout policies is that while these policies may prevent fire sales ex-post, if anticipated, they could actually increase leverage and financial instability ex-ante (Farhi and Tirole, 2012; Bianchi, 2016; Jeanne and Korinek, 2020). In our economy, anticipated bailouts do increase FIs' borrowing in any non-fundamental equilibrium – that is, they increase the supply of *private* safe assets. But paradoxically, precisely by increasing leverage, a sufficiently generous bailout policy can rule out the existence of non-fundamental equilibria.

We now assume that the government makes transfers $T^f \geq 0$ to intermediaries in the event that $p_1 = \underline{p}$, levying taxes T^f on outside investors. FIs' budget constraint when $p_1 = \underline{p}$ becomes

$$c_1^f + \underline{p}a_1^f + b^f = \underline{p}e^f + T^f$$

Otherwise the model remains unchanged. Since transfers are only made after a fall in prices, it is easy to see that they do not affect the fundamental equilibrium. In a non-fundamental equilibrium, clearly large enough *unanticipated* transfers could prevent a fire sale, since they allow FIs to repay debt without selling all their trees even if the price is low.[18] However, if FIs anticipate at date 0 that they will have a positive net worth following a fall in prices, they will exploit this to take on additional leverage, borrowing against the value of the bailout, i.e., the constraint on their borrowing is now:

$$b^f \leq \underline{p}e^f + T^f$$

[18] Formally, suppose that FIs have borrowed $b^f = \underline{p}e^f$, as in the non-fundamental equilibrium with some $\lambda > 0$. At the beginning of date 1, the government makes an unanticipated announcement that it will make a price-contingent transfer of $T(p_1) = (1 - p_1)b^f$ to FIs. This ensures that $p_1 = 1$, since for any p_1, FIs have net worth $p_1 e^f - b^f + T(p_1) = p_1(e^f - b^f)$, ensuring that they can always buy $e^f - b^f$ trees. This in turn ensures that $p_1 = 1$ since $e^f - b^f \geq 1 - \overline{e}$.

and this will hold with equality. That is, expectations of a higher bailout T^f increases the leverage that FIs take on.

Lemma 5. *In any non-fundamental equilibrium with a given $\lambda > 0$, debt issued by FIs b^f is increasing in T^f as long as the non-fundamental equilibrium exists.*

Proof. See Appendix D.1. □

While increased debt issuance by FIs prevents bailouts from eliminating fire sales ex-post, it does increase the quantity of safe assets held by households in any non-fundamental equilibrium. Thus, it has the same effect on asset prices as the increase in public safe assets, described above. Indeed, household consumption when $p_1 = \underline{p}$ is given by the same expression $\underline{p} + T^f$. In equilibrium, bailouts increase households' consumption following a fall in prices, since the taxes raised from outside investors pass through FIs to households. This reduces risk premia and thus reduces the return that FIs earn from borrowing against the value of the bailout to such an extent that they would be left with zero net worth in the event of a fall in prices. If the anticipated bailout is large enough, fear of a fall in prices at date 1 can no longer be self-fulfilling. Even if FIs and households expected prices to fall at date 1 with some probability, FIs would not wish to take on so much leverage that they would be left with zero net worth even after a generous bailout, since the excess return to this strategy would be too low. Thus, FIs would not be exposed to the risk of a fire sale and hence the fear of a fall in prices cannot be self-confirming. That is, a large enough bailout rules out non-fundamental equilibria.

Proposition 6. *If the government makes a transfer T^f to FIs which satisfies $T^f \geq \underline{p}^{\frac{1}{\gamma}} \left(1 - \chi_0^f\right) - \underline{p}$, then no non-fundamental equilibrium exists and the only equilibrium is the fundamental equilibrium described in Proposition 1.*

Proof. See Appendix D. □

Transfers to intermediaries can also be interpreted as a partial government guarantee on privately issued safe assets: the government announces that it will guarantee FIs against losses up to T^f. For example, on September 19, 2008 the US Treasury announced that it would guarantee all money market mutual funds against losses (although this announcement was intended to stop the run on money market funds rather than to prevent fire sales). Under this interpretation, the portion of FIs' liabilities that are covered by the guarantee can be thought of as *publicly backed* private safe assets. An increase in the supply of these publicly backed private safe assets has much the same effect as the increase in public safe assets, as described above. While the government guarantee does not *directly* prevent intermediaries from issuing additional debt that is not covered by the guarantee, a large enough increase in guaranteed private debt crowds out unbacked debt, reducing risk premia and eliminating FIs' incentive to take extremely risky positions. Thus, while government debt issuance and bailouts might superficially seem like quite different policies, they prevent non-fundamental price volatility in a similar way: by exploiting the government's fiscal capacity to increase the quantity of publicly backed safe assets held by households.[19]

Our result is reminiscent of Bocola and Lorenzoni (2020)'s result that government accumulation of foreign currency reserves need not induce the private sector to take riskier positions ex-ante. In their open economy environment, when the government can credibly rule out currency crises, it reduces domestic currency interest rates ex-ante. This discourages banks from borrowing in foreign currency and leaves them less exposed to a crisis. Similarly, in our economy, when a bailout or government debt issuance rules out non-fundamental price volatility, it reduces risk premia and discourages FIs from taking excessive leverage ex-ante.

There is also an interesting contrast between our results and those of Farhi and Tirole (2012). They find that making transfers to distressed financial institutions in crises makes intermediaries' leverage decisions strategic complements: if only a few banks take on leverage, it is not profitable to take on risk since a bailout is unlikely even after an adverse shock, while if many banks take on leverage, policymakers will be forced to intervene after a crisis and it is privately optimal to take on risk in anticipation of the bailout. In a sense, our result is the opposite of this: multiple equilibria and non-fundamental volatility exist absent commitment to a bailout policy while a sufficiently large bailout can eliminate this multiplicity. The reason for this difference is that in our economy, the profitability of taking on leveraged positions depends on risk-averse households' demand for safe assets. Even without a bailout, socially excessive risk taking can be privately profitable because households are willing to pay a premium to take the other side of these bets – but only if intermediaries take on so much risk that fire sales can arise. By the same token, a large enough increase in the supply of publicly backed safe assets can satiate household demand and make it unprofitable for intermediaries to take extremely risky positions, ruling out financial fragility. This relation between the quantity of safe assets held by households and the risk premium is absent in Farhi and Tirole (2012)'s economy where all agents are risk-neutral.

[19] Benigno and Robatto (2019) find a similar equivalence result in a model with exogenous risk: deposit insurance is equivalent to an increase in the supply of public safe assets because the consolidated balance sheet of all agents that supply liquidity, i.e., the government and intermediaries, is identical under the two policies.

4.2 Reducing the demand for safe assets

Transfers to households We have seen that the key to preventing non-fundamental price volatility is to reduce risk premia to such an extent that intermediaries no longer find it profitable to take extremely leveraged positions in trees. Rather than increasing the quantity of publicly backed safe assets, a direct way of reducing risk premia is to provide insurance to households. We now assume that rather than making transfers to FIs, the government makes transfers to households $T^h \geq 0$ in the event that $p_1 = \underline{p}$, levying taxes $\underline{T} = T^h$ on outside investors. Households' budget constraint when $p_1 = \underline{p}$ becomes

$$c_1^h = \underline{p}e^h + b^h + T^h$$

Otherwise the model remains unchanged. The following proposition shows that sufficiently large transfers to households rule out non-fundamental equilibria.

Proposition 7. *If the government makes a transfer T^h to households which satisfies $T^h \geq \frac{\underline{p}^{\frac{1}{\gamma}}\left(1-\chi_0^f\right)-\underline{p}}{1+\underline{p}^{\frac{1}{\gamma}}}$, then no non-fundamental equilibrium exists and the only equilibrium is the fundamental equilibrium. This minimum required transfer to households is smaller than the minimum required transfer to FIs defined in Proposition 6.*

Proof. See Appendix E. □

Promising transfers to households in the event of a fall in prices naturally reduces households' demand for safe assets, reducing the spread between risky trees and safe bonds. Large enough transfers discourage the build up of intermediary leverage and hence eliminate fire sales. This policy, like the others discussed so far, relies on the government's ability to levy taxes, even though no taxes need actually be raised in equilibrium. However, there is a sense in which transfers to households require less fiscal capacity (and therefore may be more credible) than transfers to FIs. Note that the minimum transfer to households required to eliminate all non-fundamental equilibria (described in Proposition 7) is smaller than the minimum transfer to FIs (described in Proposition 6). Suppose that the government can only raise tax revenues up to some maximum amount $T_{\max}$. If $\frac{\underline{p}^{\frac{1}{\gamma}}\left(1-\chi_0^f\right)-\underline{p}}{1+\underline{p}^{\frac{1}{\gamma}}} < T_{\max} < \underline{p}^{\frac{1}{\gamma}}\left(1-\chi_0^f\right) - \underline{p}$, then it is possible to prevent non-fundamental equilibria by promising transfers to households but not by promising transfers to intermediaries. Intuitively, this is because transfers to intermediaries increase private debt issuance and raise households' consumption in both states of the world at date 1, whereas transfers to households only increase their consumption following a fall in prices. Since the demand for safe assets depends on households' relative consumption in the two states, this means that a dollar of transfers has a greater effect on the price of all assets when given to households, rather than intermediaries.

Market maker of last resort Rather than make transfers to households following a fall in prices, a government could use its fiscal capacity to mitigate such a fall in prices by standing ready to buy any quantity of trees at some price $p^\diamond > \underline{p}$. Such a commitment is similar to many policies that have been adopted in episodes of financial distress, e.g., the ECB's Outright Monetary Transactions (a commitment to purchase Euro area sovereign bonds in potentially unlimited quantities as necessary to reduce sovereign spreads), the various facilities introduced by the Federal Reserve (e.g., the Municipal Liquidity Facility and the Secondary Market Corporate Credit Facility, which involved commitments to purchase municipal and corporate debt, respectively) and most recently the Federal Reserve's standing repo facilities (a commitment to conduct overnight reverse repo operations with a specified minimum bid-rate). As is the case with most of these examples, we will assume that the price at which the government stands ready to buy is discounted relative to the asset's fundamental value, i.e., $p^\diamond < 1$. The government raises taxes T on outside investors to fund any such purchases, and for simplicity we assume that they do not consume or distribute the apples produced by the trees they buy. Formally, we have

$$\begin{aligned} p_1 e_1^g &= T && (43) \\ p_1 \geq p^\diamond, \qquad e_1^g &\geq 0, \qquad \text{with at least one equality} && (44) \\ a_1^f + a_1^o + e_1^g &= 1 && (45) \end{aligned}$$

Clearly, this commitment does not change prices or allocations in the fundamental equilibrium since the government does not intervene when $p_1 = 1$. In any non-fundamental equilibrium, the price cannot fall below $p^\diamond$. That is, in the event of a fall in prices, the government (rather than outside investors) becomes the marginal buyer of trees, purchasing $e^g = 1 - v'^{-1}(p^\diamond)$ trees and levying taxes $T = p^\diamond e^g$ on the outside investors. The same equations characterize non-fundamental equilibria as in our baseline economy except that $\underline{p}$ is replaced by $p^\diamond$. A higher price $p^\diamond$ reduces the risk that households face and limits their demand for insurance – effectively the government provides a certain amount of insurance at a zero price. Since holding trees no longer bears as much risk, households are less willing to sell trees at a low price and buy insurance at a high price. This in turn reduces the incentives of the FIs to make levered purchases of trees which leaves them exposed to fire sales. Consequently, a high enough price floor eliminates the fire sales and hence the non-fundamental equilibrium. Importantly, a commitment to buy trees at a price $p^\diamond > \underline{p}$ can prevent a fall in prices, even though in equilibrium it is never necessary to buy assets at $p^\diamond$. Similarly, the ECB's announcement of OMT is widely credited with stabilizing sovereign spreads, even though the program was never used.

Proposition 8. *If the government commits to buy trees at a price $p^\diamond \geq \left(1-\chi_0^f\right)^{\frac{\gamma}{\gamma-1}}$, then no non-fundamental equilibrium exists and only the fundamental equilibrium described in Proposition 1 exists.*

Proof. The equations characterizing the non-fundamental equilibrium are the same as those in the baseline economy, replacing $\underline{p}$ with $p^\diamond$. In the baseline economy, non-fundamental equilibria only exist when $\chi_0^f < 1-\underline{p}^{\frac{\gamma-1}{\gamma}}$. It follows immediately that no non-fundamental equilibrium exists when $p^\diamond \geq \left(1-\chi_0^f\right)^{\frac{\gamma}{\gamma-1}}$. □

As with the other policies, fiscal capacity underpins the government's ability to eliminate non-fundamental price fluctuations with a price floor, even though it is never necessary to raise taxes in equilibrium.

Finally, it is instructive to contrast our paper with the seminal framework of Diamond and Dybvig (1983).[20] In the Diamond and Dybvig (1983) framework, financial intermediaries perform a useful role – helping households insure against (idiosyncratic) fundamental risk – but this introduces financial fragility, i.e. the risk of self-fulfilling runs. Public backstops (in the form of deposit insurance) remove fragility, facilitating intermediaries' provision of risk sharing. In our framework, financial intermediation is not intrinsically useful: the risk that intermediaries provide insurance against is itself endogenously generated by the financial sector. Public backstops remove fragility, not by helping intermediaries perform their role, but by removing the need for intermediation altogether.

5 Conclusion

In this paper we have explored how a demand for insurance can perversely help generate the risks that investors seek to insure themselves against. The fear of a fall in asset prices induces investors to hedge against these risks by buying put options or exchanging their risky assets for non-state-contingent bonds. But the financial intermediaries who sell these assets use the proceeds to take leveraged positions in the risky asset, leaving them vulnerable to self-fulfilling fire sales. Government intervention can prevent such self-fulfilling non-fundamental fluctuations by issuing public safe assets to crowd out private financial intermediation, or by committing to make transfers to intermediaries or households, or to buy risky assets at a

[20]We thank Anatoli Segura for suggesting this comparison.

modest discount to their fundamental value – commitments that never need to be exercised in equilibrium.

Much of the literature on financial crisis studies how leverage and fire sales can amplify the impact of exogenous fundamental shocks hitting the economy. We have deliberately taken a alternative approach in which the financial system does not merely amplify risk, it actually *generates* risk in an otherwise fundamentally safe economy. Clearly, in reality, both fundamental and non-fundamental risk may contribute to financial fragility. Just as the feedback loop between insurance and price volatility that we have studied can generate non-fundamental risk, it would also amplify the economy's response to fundamental shocks.

References

Acharya, Sushant and Keshav Dogra, "The Side Effects of Safe Asset Creation," *Journal of the European Economic Association*, 2021.

Adrian, Tobias and Hyun Song Shin, "Procyclical Leverage and Value-at-Risk," *The Review of Financial Studies*, 10 2013, *27* (2), 373–403.

_ **and Nina Boyarchenko**, "Intermediary Leverage Cycles and Financial Stability," *FRB of New York Staff Report no. 567*, 2012.

Allen, Franklin and Douglas Gale, "Limited Market Participation and Volatility of Asset Prices," *The American Economic Review*, 1994, pp. 933–955.

Balasko, Yves, "Extrinsic uncertainty revisited," *Journal of Economic Theory*, December 1983, *31* (2), 203–210.

Barro, Robert J., Jesús Fernández-Villaverde, Oren Levintal, and Andrew Mollerus, "Safe Assets," NBER Working Papers 20652, National Bureau of Economic Research, Inc October 2014.

Basak, Suleyman, "A general equilibrium model of portfolio insurance," *The Review of Financial Studies*, 1995, *8* (4), 1059–1090.

Benigno, Pierpaolo and Roberto Robatto, "Private Money Creation, Liquidity Crises, and Government Interventions," *Journal of Monetary Economics*, 2019, *106*, 42–58.

Bianchi, Javier, "Overborrowing and Systemic Externalities in the Business Cycle," *American Economic Review*, December 2011, *101* (7), 3400–3426.

—, "Efficient Bailouts?," *American Economic Review*, December 2016, *106* (12), 3607–59.

Bocola, Luigi and Guido Lorenzoni, "Financial crises, dollarization, and lending of last resort in open economies," *American Economic Review*, 2020, *110* (8), 2524–57.

Bowman, David and Jon Faust, "Options, Sunspots, and the Creation of Uncertainty," *Journal of Political Economy*, October 1997, *105* (5), 957–975.

Brennan, Michael J and Eduardo S Schwartz, "Portfolio insurance and financial market equilibrium," *Journal of Business*, 1989, pp. 455–472.

Brunnermeier, Markus K and Lasse Heje Pedersen, "Market liquidity and funding liquidity," *The Review of Financial Studies*, 2009, *22* (6), 2201–2238.

Buiter, Willem and Anne Sibert, "The central bank as the market-maker of last resort: from lender of last resort to market-maker of last resort," *The First Global Financial Crisis of the 21st Century*, 2008, *171.*

Caballero, Ricardo J, "On the Macroeconomics of Asset Shortages," in "The Role of Money: Money and Monetary Policy in the Twenty-First Century," Vol. 4 2006, pp. 272–283.

— **and Emmanuel Farhi**, "The Safety Trap," *The Review of Economic Studies*, 2018, *85* (1), 223–274.

— **, — , and Pierre-Olivier Gourinchas**, "The safe assets shortage conundrum," *Journal of Economic Perspectives*, 2017, *31* (3), 29–46.

Cass, David and Karl Shell, "Do Sunspots Matter?," *Journal of Political Economy*, April 1983, *91* (2), 193–227.

Cheng, Jeffrey, David Wessel, and Joshua Younger, "How did COVID-19 disrupt the market for US Treasury debt?," *Hutchins Center, Brookings Institution, May 2020*, 2020.

Chowdhry, Bhagwan and Vikram Nanda, "Leverage and market stability: The role of margin rules and price limits," *The Journal of Business*, 1998, *71* (2), 179–210.

Danielsson, Jon and Hyun Song Shin, "Endogenous risk," *Modern risk management: A history*, 2003, pp. 297–316.

Dávila, Eduardo and Anton Korinek, "Pecuniary externalities in economies with financial frictions," *The Review of Economic Studies*, 2018, *85* (1), 352–395.

DeLong, J Bradford, Andrei Shleifer, Lawrence H Summers, and Robert J Waldmann, "Positive Feedback Investment Strategies and Destabilizing Rational Speculation," *Journal of Finance*, 1990, *45* (2), 379–395.

Diamond, Douglas W and Philip H Dybvig, "Bank runs, deposit insurance, and liquidity," *Journal of political economy*, 1983, *91* (3), 401–419.

Diamond, William, "Safety Transformation and the Structure of the Financial System," *Journal of Finance*, December 2020, *75* (6), 2973–3012.

Farhi, Emmanuel and Jean Tirole, "Collective Moral Hazard, Maturity Mismatch, and Systemic Bailouts," *American Economic Review*, February 2012, *102* (1), 60–93.

Gennaioli, Nicola, Andrei Shleifer, and Robert Vishny, "Neglected risks, financial innovation, and financial fragility," *Journal of Financial Economics*, 2012, *104* (3), 452–468. Market Institutions, Financial Market Risks and Financial Crisis.

Gennotte, Gerard and Hayne Leland, "Market liquidity, hedging, and crashes," *The American Economic Review*, 1990, pp. 999–1021.

Gorton, Gary B and Guillermo Ordoñez, "The Supply and Demand for Safe Assets," *Available at SSRN 2202237*, 2020.

Gottardi, Piero and Atsushi Kajii, "The Structure of Sunspot Equilibria: The Role of Multiplicity," *The Review of Economic Studies*, 1999, *66* (3), 713–732.

_ , Vincent Maurin, and Cyril Monnet, "Financial Fragility with Collateral Circulation," 2021.

Greenwood, Robin, Samuel G. Hanson, and Jeremy C. Stein, "The Federal Reserve's Balance Sheet as a Financial-Stability Tool," *2016 Economic Policy Symposium Proceedings*, 2016.

Grossman, Sanford J, "An Analysis of the Implications for Stock and Futures Price Volatility of Program Trading and Dynamic Hedging Strategies," *The Journal of Business*, 1988, *61* (3), 275–298.

_ and Zhongquan Zhou, "Equilibrium analysis of portfolio insurance," *The Journal of Finance*, 1996, *51* (4), 1379–1403.

Gu, Chao, Cyril Monnet, Ed Nosal, and Randall Wright, "On the instability of banking and other financial intermediation," 2020.

He, Zhiguo and Arvind Krishnamurthy, "Intermediary asset pricing," *American Economic Review*, 2013, *103* (2), 732–70.

Hens, Thorsten, "Do sunspots matter when spot market equilibria are unique?," *Econometrica*, 2000, *68* (2), 435–441.

Infante, Sebastian and Guillermo Ordoñez, "The Collateral Link between Volatility and Risk Sharing," Technical Report, National Bureau of Economic Research 2020.

Jacklin, Charles J, Allan W Kleidon, and Paul Pfeiderer, "Underestimation of portfolio insurance and the crash of October 1987," *The Review of Financial Studies*, 1992, *5* (1), 35–63.

Jeanne, Olivier and Anton Korinek, "Macroprudential Regulation versus mopping up after the crash," *The Review of Economic Studies*, 02 2020, *87* (3), 1470–1497.

Kiyotaki, Nobuhiro and John Moore, "Credit cycles," *Journal of political economy*, 1997, *105* (2), 211–248.

Krishnamurthy, Arvind, "Collateral constraints and the amplification mechanism," *Journal of Economic Theory*, 2003, *111* (2), 277–292.

_ **and Annette Vissing-Jorgensen**, "The Aggregate Demand for Treasury Debt," *Journal of Political Economy*, 2012, *120* (2), 233–267.

_ **and Wenhao Li**, "Dissecting mechanisms of financial crises: Intermediation and sentiment," Technical Report, National Bureau of Economic Research 2020.

Lorenzoni, Guido, "Inefficient Credit Booms," *The Review of Economic Studies*, 2008, *75* (3), 809–833.

Malherbe, Frédéric, "Self-fulfilling Liquidity Dry-ups," *The Journal of Finance*, 2014, *69* (2), 947–970.

Mas-Colell, Andreu, "Three observations on sunspots and asset redundancy," in "Economic Analysis of Markets and Games : Essays in Honor of Frank Hahn," Cambridge, Mass.: MIT Press, 1992, pp. 465–474.

Negro, Marco Del, Gauti Eggertsson, Andrea Ferrero, and Nobuhiro Kiyotaki, "The great escape? A quantitative evaluation of the Fed's liquidity facilities," *American Economic Review*, 2017, *107* (3), 824–57.

Oehmke, Martin, "Liquidating illiquid collateral," *Journal of Economic Theory*, 2014, *149*, 183–210.

Rajan, Raghuram G, "Has Financial Development Made the World Riskier?," 2005. presented in the session, Financial Markets, Financial Fragility, and Central Banking, at "The Greenspan Era: Lessons for the Future" symposium, sponsored by the Fed of Kansas City, at Jackson Hole, Wyoming, August 25-27.

Schmitt-Grohé, Stephanie and Martín Uribe, "Multiple Equilibria in Open Economies with Collateral Constraints," *The Review of Economic Studies*, 05 2020, *88* (2), 969–1001.

Segura, Anatoli and Alonso Villacorta, "The paradox of safe asset creation," *Bank of Italy Temi di Discussione (Working Paper) No*, 2020, *1260.*

Shin, Hyun Song, *Risk and liquidity*, Oxford University Press, 2010.

Shleifer, Andrei and Robert W Vishny, "Liquidation values and debt capacity: A market equilibrium approach," *The journal of finance*, 1992, *47* (4), 1343–1366.

_ **and** _ , "The limits of arbitrage," *The Journal of finance*, 1997, *52* (1), 35–55.

_ **and** _ , "Fire sales in finance and macroeconomics," *Journal of Economic Perspectives*, 2011, *25* (1), 29–48.

Simsek, Alp, "Speculation and Risk Sharing with New Financial Assets," *The Quarterly Journal of Economics*, 05 2013, *128* (3), 1365–1396.

Stein, Jeremy C, "Monetary policy as financial stability regulation," *The Quarterly Journal of Economics*, 2012, *127* (1), 57–95.

Svensson, Lars E O, "Efficiency and Speculation in a Model with Price-Contingent Contracts," *Econometrica*, January 1981, *49* (1), 131–151.

Appendix

A Detailed Derivation: Repo Contracts

In this section we provide a detailed derivation of Proposition 4 presented in Section 3. We construct an equilibrium in which $p_1 = 1$ with probability $1 - \lambda$ and equals $\underline{p} = v'(1) < 1$ with probability λ. $\lambda \in (0, 1)$ is the probability of a sunspot which results in a low realization of the price of trees at date 1. Furthermore the issuance constraint on FIs (36) binds when $p_1 = \underline{p}$ is realized:

$$b^f = \underline{p}e^f \qquad \Rightarrow \qquad \frac{b^f}{e^f} = \underline{p} \tag{46}$$

First we show that this is a date 1 equilibrium. Given that $p_1 = \underline{p}$, FI's must sell all their trees to pay out on the debt they issued. Thus, in equilibrium, all trees must be purchased by outside investors who are only willing to pay a price of $v'(1) < 1$ for all these trees - confirming the lower price.

Given that this is a non-state contingent contract, FIs need to sell b^f trees even when $p_1 = 1$. For this to be an equilibrium, we need to show that

$$b^f \leq \bar{e} - e^h \iff \underline{p}(1 - e^h) \leq \bar{e} - e^h. \tag{47}$$

That is, the number of trees sold to outside investors are below $\bar{e}$ so that price of one can be sustained as an equilibrium. This condition reduces to $\underline{p}(1 - e^h) \leq \bar{e} - e^h$. We will later show that this constraint is not a binding restriction in equilibrium.[21]

Given the equilibrium at date 1, next we describe the equilibrium behavior of agents at date 0. Given equation (46), the date 0 problem of FIs can be written as

$$\max_{e^f, b^f} \chi_0^f - p_0 e^f + q b^f + \mathbb{E}\left[e^f - \frac{1}{p_1} b^f\right]$$

s.t.

$$\begin{aligned} \chi_0^f - p_0 e^f + q b^f &\geq 0 & (48) \\ b^f &\leq \underline{p} e^f & (49) \end{aligned}$$

[21] The constraint can be rewritten as $e^f \geq \frac{1-\bar{e}}{1-\underline{p}} \leq 1$. The maximum quantity of trees that the FIs can purchase is bounded above by one.

where the objective function uses the fact that $p_1 \leq 1$, and so, any remaining funds that the FI's have after debt repayments are used to buy trees and eat the apples they produce. The first constraint (48) is simply the non-negativity constraint on date 0 consumption. The second constraint imposes that FI's must be able to repay debt claims even after the lowest realization of $p_1 = \underline{p}$. This constraint can be written as $b^f \leq \underline{p}e^f$. When both constraints bind, we have

$$e^f = \frac{\chi_0^f}{p_0 - \underline{p}q} \geq \frac{\chi_0^f}{p_0} \tag{50}$$

That is, the maximum quantity of trees that a FI can buy (the intersection of the solid blue and dashed red lines in Figure 2) is greater than χ_0^f/p_0, the quantity they could purchase using all of their endowment (the intersection of the dashed red line and the horizontal axis in Figure 2). There is a multiplier effect: buying trees increases the amount of insurance that can be issued, issuing debt provides more funds with which to purchase trees, and so forth.

While FIs always *can* purchase $\frac{\chi_0^f}{p_0 - \underline{p}q}$ trees, whether they want to do so depends on their induced preferences over insurance and trees. It is clear that they will purchase the maximum quantity of trees if their indifference curves are steeper than $\frac{p_0}{q}$, i.e., if

$$\frac{q}{p_0} > \mathbb{E}\left[\frac{1}{p_1}\right] \tag{51}$$

We conjecture that in equilibrium this condition is satisfied; we will show that, if $\lambda > 0$ is not too large, this is indeed the case.

Next we describe optimal household behavior at date 0. Here we can use the fact that households consume $\underline{p}$ after a low realization of p_1. Their optimization problem yields the following Euler equations for trees and insurance respectively

$$p_0 = \frac{\lambda \underline{p}^{1-\gamma} + (1-\lambda)(\underline{p} + (1-\underline{p})e^h)^{-\gamma}}{\left[\lambda \underline{p}^{1-\gamma} + (1-\lambda)(\underline{p} + (1-\underline{p})e^h)^{1-\gamma}\right]^{\frac{\gamma}{\gamma-1}}} \tag{52}$$

$$q = \frac{\lambda \underline{p}^{-\gamma} + (1-\lambda)(\underline{p} + (1-\underline{p})e^h)^{-\gamma}}{\left[\lambda \underline{p}^{1-\gamma} + (1-\lambda)(\underline{p} + (1-\underline{p})e^h)^{1-\gamma}\right]^{\frac{\gamma}{\gamma-1}}} \tag{53}$$

Combining (52)-(53) we get

$$p_0 - \underline{p}q = \frac{(1-\lambda)(1-\underline{p})(\underline{p} + (1-\underline{p})e^h)^{-\gamma}}{\left[\lambda \underline{p}^{1-\gamma} + (1-\lambda)(\underline{p} + (1-\underline{p})e^h)^{1-\gamma}\right]^{\frac{\gamma}{\gamma-1}}} \tag{54}$$

Combining (54) with the FIs demand for these claims (50) and using the fact that $e^h = 1-e^f$ we have

$$\frac{(1-\lambda)(1-\underline{p})(1-e^h)(\underline{p}+(1-\underline{p})e^h)^{-\gamma}}{\left[\lambda\underline{p}^{1-\gamma}+(1-\lambda)(\underline{p}+(1-\underline{p})e^h)^{1-\gamma}\right]^{\frac{\gamma}{\gamma-1}}} = \chi_0^f \tag{55}$$

Let $x \equiv \underline{p}+(1-\underline{p})e^h$. Then, we can write the equilibrium condition as

$$\frac{(1-\lambda)(1-x)x^{-\gamma}}{\left[\lambda\underline{p}^{1-\gamma}+(1-\lambda)x^{1-\gamma}\right]^{\frac{\gamma}{\gamma-1}}} = \chi_0^f \tag{56}$$

The LHS of the expression above is strictly decreasing, so this defines a unique solution for e^h. It remains to check (51) hold, i.e., it is indeed optimal for FIs to lever up to the maximum. Using (52)-(53), (51) becomes

$$x \equiv \underline{p}+(1-\underline{p})e^h > \underline{p}^{\frac{\gamma-1}{\gamma}} \tag{57}$$

Notice that equations (56) and (57) are identical to the conditions derived in the insurance economy (see equations (29) and (30)). With some algebra, we can show that the RHS of (57) is smaller than $\underline{p}^{\frac{\gamma-1}{\gamma}}$. Consequently, (51) is satisfied if

$$e^h > \underline{p}^{\frac{\gamma-1}{\gamma}} \tag{58}$$

Since e^h is decreasing in χ_0^f, this is satisfied as long as χ_0^f is within a particular range as the Proposition 4 in Section 3 summarizes. The only restriction that remains to be verified is that (47) holds, that is the maximum quantity of bonds sold are below a limit, and Equation (47) implies the following restriction

$$x \equiv \underline{p}+(1-\underline{p})e^h \le \bar{e} \tag{59}$$

This condition imposes a lower bound on λ in Proposition 4. With some algebra, we can show that this lower bound is negative, and hence non-binding.

B Proof of Proposition 4

Take any collection of prices (p_0, q) and allocations (e^h, e^f, z^h, z^f) in the economy with insurance. Define the corresponding prices and allocations in the economy with bonds to be $\hat{p}_0 = p_0$, $\hat{q}^b = p_0+q$, $\hat{e}^h = e^h - z^h$, $\hat{e}^f = e^f + z^f$, $\hat{b}^h = z^h$, $\hat{b}^f = \hat{z}^f$. We claim that (e^h, e^f, z^h, z^f)

is feasible for both households and FIs in the economy with insurance, given (p_0, q), if and only if the corresponding allocation $(\hat{e}^h, \hat{e}^f, \hat{b}^h, \hat{b}^f)$ is feasible for both households and FIs in the economy with bonds given the corresponding prices $(\hat{p}_0, \hat{q}^b)$. To see this, note that a feasible allocation with insurance satisfies the household and FI budget constraints (15), (16), (17), (18) and the non-negativity constraint $p_1 e^f - (1-p_1)z^f \geq 0$. Substituting in the expressions for the corresponding prices and allocations, these constraints become

$$\begin{aligned} c_0^h + \hat{p}_0 \hat{e}^h + \hat{q}^b \hat{b}^h &= \chi_0^h + \hat{p}_0 \\ c_1^h &= p_1 \hat{e}^h + \hat{b}^h \\ c_0^f + \hat{p}_0 \hat{e}^f &= \chi_0^f + \hat{q}^b \hat{b}^f \\ c_1^f + p_1 a_1^f + \hat{b}^f &= p_1 \hat{e}^f \\ p_1 \hat{e}^f - \hat{b}^f &\geq 0 \end{aligned}$$

That is, the corresponding allocation is also feasible given the corresponding prices in the bond economy.

Now let $(p_0, q, e^h, e^f, z^h, z^f)$ describe an equilibrium in the economy with insurance when $p_1 = \underline{p}$ with probability λ. We claim that $(\hat{p}_0, \hat{q}^b, \hat{e}^h, \hat{e}^f, \hat{b}^h, \hat{b}^f)$ is an equilibrium in the economy with bonds. Recall that $\underline{p}e^f = (1-\underline{p})z^f$ in any equilibrium with insurance. Using the definition of the corresponding allocation in the bond equilibrium, this implies that $\underline{p}\hat{e}^f = \hat{b}^f$. In other words, when $p_1 = \underline{p}$, FIs must sell all their trees to pay out on the debt contract. Conversely, if $p_1 = 1$, the FIs budget constraint is $c_1^f + a_1^f = \hat{e}^f - \hat{b}^f = (1-\underline{p})\hat{e}^f > 0$, i.e., the FIs need not sell any trees to outside investors, sustaining $p_1 = 1$ as in the fundamental equilibrium. It also follows immediately that $\hat{e}^h, \hat{e}^f, \hat{b}^h, \hat{b}^f$ is optimal given $\hat{p}_0 = p_0$ and $\hat{q}^b$: we know this allocation is feasible (since the corresponding allocation in the insurance equilibrium is optimal and hence feasible), and it cannot be the case that any other feasible allocation delivers higher utility for either households or FIs (since that would imply that the corresponding allocation in the insurance equilibrium was not optimal). Thus, $(\hat{p}_0, \hat{q}^b, \hat{e}^h, \hat{e}^f, \hat{b}^h, \hat{b}^f)$ is indeed an equilibrium in the economy with bonds. It also follows that the consumption of all agents is the same across these two equilibria. Finally, equations (37)-(39) are obtained by appropriately transforming (26), (27) and (29) using the definition of the corresponding prices and allocations.

C Proof of Proposition 5

Throughout the proof, we characterize outcomes in a non-fundamental equilibrium, assuming that such an equilibrium exists. We show that this leads to a contradiction when $b^g \geq b^*$.

We begin by characterizing household consumption at date 1 and asset prices. When $p_1 = \underline{p}$, FIs consume zero apples and cookies and all the proceeds from selling trees are used to repay their debt, i.e., $\underline{p}e^f = b^f$. Adding this to the government's date 1 budget constraint (41) when $p_1 = \underline{p}$ and using market clearing, we have

$$\underline{T} + \underline{p}(1 - e^h) = b^h \qquad \Rightarrow \qquad \underline{c}_1^h = b^h + \underline{p}e^h = \underline{p} + \underline{T}$$

where $\underline{c}_1^h$ and $\underline{T}$ denote household consumption and the tax on outside investors, respectively, in the state when $p_1 = \underline{p}$. Household consumption when $p_1 = 1$ is simply

$$\bar{c}_1^h = e^h + b_h = \underline{c}_1^h + (1 - \underline{p})e^h = (1 - \underline{p})e^h + \underline{p} + \underline{T}$$

When characterizing asset prices, it is useful to work with the ratio of household marginal utilities when $p_1 = 1$ and $p_1 = \underline{p}$:

$$g \equiv \left(\frac{\bar{c}_1^h}{\underline{c}_1^h}\right)^{-\gamma} = \left[\frac{(1 - \underline{p})e^h}{\underline{p} + \underline{T}} + 1\right]^{-\gamma} \tag{60}$$

We can write the price of a tree, bond, and the price of an Arrow security which pays when $p_1 = 1$ as

$$p_0 = \frac{\lambda \underline{p} + (1 - \lambda)\, g}{\left[\lambda + (1 - \lambda)\, g^{\frac{\gamma - 1}{\gamma}}\right]^{-\frac{\gamma}{1-\gamma}}} \tag{61}$$

$$q^b = \frac{\lambda + (1 - \lambda)\, g}{\left[\lambda + (1 - \lambda)\, g^{\frac{\gamma - 1}{\gamma}}\right]^{\frac{\gamma}{\gamma - 1}}} \tag{62}$$

$$q^a = \frac{1}{1 - \underline{p}} p_0 - \frac{\underline{p}}{1 - \underline{p}} q^b = \frac{(1 - \lambda)\, g}{\left[\lambda + (1 - \lambda)\, g^{\frac{\gamma - 1}{\gamma}}\right]^{-\frac{\gamma}{1-\gamma}}} \tag{63}$$

Note for future reference that

$$\begin{aligned}
\frac{\partial p_0}{\partial g} &= p_0\,(1 - \lambda)\, g^{-\frac{1}{\gamma}} \left[\frac{\lambda\left(1 - \underline{p}g^{-\frac{1}{\gamma}}\right)}{\left[\lambda \underline{p} g^{-\frac{1}{\gamma}} + (1 - \lambda)\, g^{\frac{\gamma - 1}{\gamma}}\right]\left[\lambda + (1 - \lambda)\, g^{\frac{\gamma - 1}{\gamma}}\right]}\right] > 0 \\
&\because \underline{p}g^{-\frac{1}{\gamma}} = \left(1 - \underline{p}\right) \frac{\underline{p}}{\underline{p} + \underline{T}} e^h + \underline{p} < 1
\end{aligned}$$

$$\frac{\partial q^b}{\partial g} = q^b (1-\lambda) g^{-\frac{1}{\gamma}} \left[\frac{\lambda \left(1 - g^{-\frac{1}{\gamma}}\right)}{\left[\lambda g^{-\frac{1}{\gamma}} + (1-\lambda) g^{\frac{\gamma-1}{\gamma}}\right] \left[\lambda + (1-\lambda) g^{\frac{\gamma-1}{\gamma}}\right]} \right] < 0 \qquad \because g < 1$$

and $\frac{dq^a}{dg} > 0$ by inspection.

In the economy with bonds, the necessary condition in order for FIs to be willing to lever up to the maximum, and the analog of (25) is the economy with insurance, is

$$\frac{q^b}{p_0} - 1 > \mathbb{E}\left[\frac{1-p_1}{p_1}\right] = \lambda \frac{1-\underline{p}}{\underline{p}}$$

Using the above expressions for prices, this condition simplifies to $\underline{p} > g$. Next, we show that g is increasing in b^g and thus this necessary condition is violated if b^g is large enough, i.e., $b^g \geq b^*$.

To characterize equilibrium allocations as functions of b^g, we use market-clearing for trees (42). If in a non-fundamental equilibrium, FIs take on maximum leverage: $b^f = \underline{p}e^f$ and so their date 0 budget constraint becomes:

$$p_0 e^f = \chi_0^f + q^b b^f \qquad \Rightarrow \qquad e^f = \frac{\chi_0^f}{(1-\underline{p})q^a}$$

The government's demand for trees at date 0 is simply

$$e^g = \frac{q^b}{p_0} b^g$$

Finally, we need an expression for e^h in terms of b^g and g. Substituting out for e^g in the date 0 government budget constraint (40), we get

$$\underline{T} = \left[(1-\underline{p}) - \underline{p}\left(\frac{q^b}{p_0} - 1\right)\right] b^g$$

Using this together with the definition of g, we can express e^h as a function of g and b^g:

$$e^h = \frac{\left(g^{-\frac{1}{\gamma}} - 1\right)\left(\underline{p}\left[1 - \left(\frac{q^b}{p_0} - 1\right) b^g\right] + (1-\underline{p})\, b^g\right)}{1-\underline{p}}$$

Substituting for e^h, e^f and e^g in the market clearing condition for trees, we get

$$\frac{\left(g^{-\frac{1}{\gamma}} - 1\right)\left(\underline{p}\left[1 - \left(\frac{q^b}{p_0} - 1\right) b^g\right] + (1-\underline{p})\, b^g\right)}{1-\underline{p}} + \frac{\chi_0^f}{(1-\underline{p})q^a} + \frac{q^b}{p_0} b^g = 1$$

Using the expressions for p_0, q^b and q^a and rearranging, the expression above can be rewritten as

$$\mathbf{F}(g, b^g, \lambda) \equiv \frac{\chi_0^f \left[\lambda + (1-\lambda) g^{\frac{\gamma-1}{\gamma}}\right]^{-\frac{\gamma}{1-\gamma}}}{\left(1-\underline{p}\right)(1-\lambda) g} + \frac{\lambda + (1-\lambda) g^{\frac{\gamma-1}{\gamma}}}{\lambda \underline{p} + (1-\lambda) g} b^g + \frac{\underline{p}}{1-\underline{p}} \left(g^{-\frac{1}{\gamma}} - 1\right) - 1 = 0 \quad (64)$$

Next, taking the derivative of $\mathbf{F}$ w.r.t. b^g, g and λ, we get

$$\frac{\partial \mathbf{F}}{\partial b^g} = \frac{\lambda + (1-\lambda) g^{\frac{\gamma-1}{\gamma}}}{\lambda \underline{p} + (1-\lambda) g} > 0$$

$$\begin{aligned}\frac{\partial \mathbf{F}}{\partial g} &= -\frac{\lambda \chi_0^f \left[\lambda + (1-\lambda) g^{\frac{\gamma-1}{\gamma}}\right]^{\frac{1}{\gamma-1}}}{\left(1-\underline{p}\right)(1-\lambda) g^2} + \frac{(1-\lambda)}{\lambda \underline{p} + (1-\lambda) g} \left[\frac{\lambda \left[\underline{p} g^{-\frac{1}{\gamma}} \left(1-\gamma^{-1}\right) - 1\right] - (1-\lambda) \gamma^{-1} g^{\frac{\gamma-1}{\gamma}}}{\lambda \underline{p} + (1-\lambda) g}\right] b^g \\ &\quad - \frac{1}{\gamma} \frac{\underline{p}}{1-\underline{p}} g^{-\frac{1+\gamma}{\gamma}} < 0\end{aligned}$$

$$\begin{aligned}\frac{\partial \mathbf{F}}{\partial \lambda} &= \frac{\chi_0^f}{\left(1-\underline{p}\right) g} \frac{\left\{\lambda + (1-\lambda) g^{\frac{\gamma-1}{\gamma}}\right\}^{-\frac{1}{1-\gamma}}}{1-\lambda} \left[\frac{\gamma}{\gamma-1} \left(1 - g^{\frac{\gamma-1}{\gamma}}\right) + \frac{\lambda + (1-\lambda) g^{\frac{\gamma-1}{\gamma}}}{(1-\lambda)}\right] \\ &\quad + \frac{g}{\lambda \underline{p} + (1-\lambda) g} \left[\frac{1 - \underline{p} g^{-\frac{1}{\gamma}}}{\lambda \underline{p} + (1-\lambda) g}\right] b^g > 0,\end{aligned}$$

where the second inequality uses the fact that $\underline{p} g^{-\frac{1}{\gamma}} = \left(1-\underline{p}\right) \frac{\underline{p}}{\underline{p}+\underline{T}} e^h + \underline{p} < 1$. Thus, by the implicit function theorem, g is increasing in both b^g and λ. Next, we characterize the value of b^g that is just sufficient to implement $g = \underline{p}$ (implying that FIs do not strictly prefer to lever up to the maximum) when $\lambda = 0$. Setting $\lambda = 0$ and $g = \underline{p}$ in (64) yields

$$b^* = \frac{\underline{p}^{\frac{1}{\gamma}}}{1-\underline{p}} \left(1 - \chi_0^f\right) - \frac{\underline{p}}{1-\underline{p}}$$

Given the comparative static results just stated, it follows that $g > \underline{p}$ whenever $\lambda > 0$ and $b^g \geq b^*$. This implies that FIs would strictly prefer not to lever up to the maximum, contradicting our assumption that a non-fundamental equilibrium exists.

D FI Bailout: Proof of Proposition 6

The proof closely follows the proof of Proposition 5. We characterize outcomes in a non-fundamental equilibrium, assuming that such an equilibrium exists, and show that this leads to a contradiction when $\underline{T} \geq \underline{p}^{\frac{1}{\gamma}} \left(1 - \chi_0^f\right) - \underline{p}$ in the state when $p_1 = \underline{p}$.

When $p_1 = \underline{p}$, FIs consume zero apples and cookies and all the proceeds from selling trees together with the government transfer $\underline{T}$ are used to repay their debt, i.e., $\underline{p}e^f + \underline{T} = b^f$. Using this condition along with market clearing, households' date 1 budget constraint when $p_1 = \underline{p}$ can be written as

$$\underline{c}_1^h = \underline{p}e^h + b^h = \underline{p} + \underline{T}$$

Household consumption when $p_1 = 1$ is simply

$$\overline{c}_1^h = e^h + b_h = \underline{c}_1^h + (1 - \underline{p})e^h = (1 - \underline{p})e^h + \underline{p} + \underline{T}$$

Since households' consumption in both states of the world at date 1 is given by the same expressions as in the economy with government debt, we can again characterize asset prices in terms of the ratio of marginal utilities in the two states g, defined in (60). As shown in Appendix C, we know that $\underline{p} > g$ is a necessary condition in order for FIs to lever up to the maximum and for a non-fundamental equilibrium to exist. In any non-fundamental equilibrium, FIs have zero consumption when $p_1 = \underline{p}$ and the date 1 budget constraint can be written as

$$\underline{p}e^f + \underline{T} = b^f$$

Substituting this into the date 0 FI budget constraint, their purchase of trees can be expressed as

$$e^f = \frac{\chi_0^f + q^b \underline{T}}{p_0 - q^b \underline{p}}$$

Substituting this into the market clearing expression for trees and using the expressions for the market clearing date 0 prices of trees and bonds (61)-(63), we have

$$\mathbf{H}(g, \underline{T}, \lambda) = \frac{\left(\underline{p} + \underline{T}\right)\left(g^{-\frac{1}{\gamma}} - 1\right)}{1 - \underline{p}} + \frac{\lambda + (1 - \lambda)\, g}{(1 - \lambda)\left(1 - \underline{p}\right) g}\underline{T} + \frac{\left[\lambda g^{-\frac{\gamma - 1}{\gamma}} + (1 - \lambda)\right]^{\frac{\gamma}{\gamma - 1}}}{(1 - \lambda)\left(1 - \underline{p}\right)}\chi_0^f - 1 = 0$$

By inspection, $\frac{\partial \mathbf{H}}{\partial g} < 0$, $\frac{\partial \mathbf{H}}{\partial \underline{T}} > 0$ and $\frac{\partial \mathbf{H}}{\partial \lambda} > 0$. Thus, by the implicit function theorem, g is increasing in both $\underline{T}$ and λ. The value of $\underline{T}$ that is just sufficient to implement $g = \underline{p}$ when $\lambda = 0$ is

$$\underline{T} = \left(1 - \chi_0^f\right)\underline{p}^{\frac{1}{\gamma}} - \underline{p}$$

Given the comparative static results just stated, it follows that $g > \underline{p}$ whenever $\lambda > 0$ and $\underline{T} \geq \left(1 - \chi_0^f\right) \underline{p}^{\frac{1}{\gamma}} - \underline{p}$. This implies that FIs would strictly prefer not to lever up to the maximum, contradicting our assumption that a non-fundamental equilibrium exists.

D.1 Proof of Lemma 5

Suppose by contradiction that b^f is not increasing in T. We know that g is increasing in T. Since $\frac{dp_0}{dg} > 0$ and $\frac{dq^b}{dg} < 0$, we also know that p_0 is increasing in T and q^b is decreasing in T. Thus, if b^f is not increasing, then $q^b b^f$ is strictly decreasing. Then the date 0 budget constraint of FIs implies that

$$e^f = \frac{\chi_0^f + q^b b^f}{p_0},$$

that is, e^f is decreasing in T. Using market clearing, this implies that e^h is increasing and b^h is decreasing. Recall that we can write g as

$$g = \left(\frac{e^h + b^h}{\underline{p}e^h + b^h}\right)^{-\gamma}$$

If e^h is increasing and b^h is decreasing in T, this expression implies that g is decreasing in T, which is a contradiction. Thus, it must be that b^f must be increasing in T.

E Transfer to households: Proof of Proposition 7

Now suppose that the government makes transfers to households when $p_1 = \underline{p}$. The proof closely follows the proof of Proposition 5. We characterize outcomes in a non-fundamental equilibrium, assuming that such an equilibrium exists, and show that this leads to a contradiction when $\underline{T} \geq \underline{p}^{\frac{1}{\gamma}} \left(1 - \chi_0^f\right) - \underline{p}$ in the state when $p_1 = \underline{p}$.

When $p_1 = \underline{p}$, FIs sell all their trees to outside investors to pay out on their liabilities to the households, who sell their own holding of trees and receive transfers $\underline{T}$ from the government. Their consumption in this state is $\underline{c}_1^h = \underline{p} + \underline{T}$. When $p_1 = 1$, households receive a higher return on their own holdings of trees, but do not receive transfers. Their consumption in this state is $\bar{c}_1^h = \underline{c}_1^h + (1 - \underline{p})e^h - \underline{T} = (1 - \underline{p})e^h + \underline{p}$. Thus the ratio of marginal utilities in the two states g is now given by

$$g = \left(\frac{(1 - \underline{p})e^h + \underline{p}}{\underline{p} + \underline{T}}\right)^{-\gamma}$$

The necessary condition for a non-fundamental equilibrium to exist is still $\underline{p} > g$, since the expressions describing asset prices (61)-(63) remain the same conditional on g. Following the same steps as in the proof above and noting that the mapping between g and e^h is now different, we can rewrite the tree market clearing condition as

$$\mathbf{G}(g, \underline{T}, \lambda) = \frac{\left(\underline{p} + T\right)}{1 - \underline{p}} g^{-\frac{1}{\gamma}} - \frac{\underline{p}}{1 - \underline{p}} + \frac{\lambda + (1 - \lambda) g}{(1 - \lambda)\left(1 - \underline{p}\right) g} T + \frac{\left[\lambda g^{-\frac{\gamma - 1}{\gamma}} + (1 - \lambda)\right]^{\frac{\gamma}{\gamma - 1}}}{(1 - \lambda)\left(1 - \underline{p}\right)} \chi_0^f - 1 = 0$$

By inspection, $\frac{\partial \mathbf{G}}{\partial g} < 0$, $\frac{\partial \mathbf{G}}{\partial \underline{T}} > 0$ and $\frac{\partial \mathbf{G}}{\partial \lambda} > 0$. Thus, by the implicit function theorem, g is increasing in both $\underline{T}$ and λ. The value of $\underline{T}$ that is just sufficient to implement $g = \underline{p}$ when $\lambda = 0$ is

$$\underline{T} = \frac{\left(1 - \chi_0^f\right) \underline{p}^{\frac{1}{\gamma}} - \underline{p}}{1 + \underline{p}^{\frac{1}{\gamma}}}$$

Following the same argument as in the previous proof, it follows that no non-fundamental equilibrium exists when $T \geq \underline{T}$.

F Market Incompleteness

Proposition 9. *If we allow outside investors to participate in asset markets at date 0, then the fundamental equilibrium is the unique equilibrium.*

Proof. If outside investors can participate at date 0, their problem can be written as

$$\max_{e^o, z^o, c_1^o, a_1^o} \mathbb{E}\left[v(a_1) + c_1^o\right]$$

subject to:

$$p_0 e^o = q z^o \tag{65}$$

$$c_1^o + p_1 a_1^o + (1 - p_1) z^o = \chi_1 + p_1 e^o \tag{66}$$

$$(1 - p_1) z^o \leq \chi_1 + p_1 e^o \tag{67}$$

where e^o denote the OI's purchase of trees at date 0 and z^o denotes the OI's issuance of insurance. As in the baseline model, we assume that χ_1 is large enough so that the OI has non-negative consumption of cookies, i.e., (67) never holds with an equality. Attaching

Lagrange multipliers μ_0 and μ_1 to (65) and (66) respectively, the first-order conditions are

$$-\mu_0 p_0 + \mathbb{E}\mu_1 p_1 = 0 \tag{68}$$

$$\mu_0 q - \mathbb{E}\mu_1(1-p_1) = 0 \tag{69}$$

$$1 - \mu_1 = 0 \tag{70}$$

$$v'(a_1^o) - \mu_1 p_1 = 0 \tag{71}$$

From the FOC with respect to (70), we have $\mu_1 = 1$. Combining the FOC with respect to e^o (equation (68)) and the FOC with respect to z^o (equation (69)) and using $\mu_1 = 1$, it must be that the expected returns are equalized on the two assets:

$$\frac{\mathbb{E}p_1}{p_0} = \frac{\mathbb{E}(1-p_1)}{q}$$

Rewriting, we have

$$\frac{q}{p_0} = \frac{1-\mathbb{E}p_1}{\mathbb{E}p_1}$$

Since $\frac{1-p}{p}$ is a convex function of p, from Jensen's inequality, it follows that

$$\frac{q}{p_0} = \frac{1-\mathbb{E}p_1}{\mathbb{E}p_1} \leq \mathbb{E}\left[\frac{1-p_1}{p_1}\right],$$

where the inequality is strict unless p_1 is degenerate. Recall from (25) that for FIs to issue insurance in equilibrium, we must have

$$\frac{q}{p_0} \geq \mathbb{E}\left[\frac{1-p_1}{p_1}\right]$$

Thus, we cannot have an equilibrium in which FIs issue insurance and p_1 is stochastic. It follows that the only equilibrium is the fundamental equilibrium. $\square$