Discussion of

Diagnostic Business Cycles

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What this paper is about?

- develop tools to incorporate Diagnostic Expectations (DE) in linear GE economies
  1. endogenous states $\rightarrow$ endogenous amplification
  2. selective memory recall based on *distant* past $\rightarrow$ boom/bust cycle

- estimate a medium-scale DSGE model with DE
  (Christiano Eichenbaum Evans 2005, Christiano Trabandt Walentin 2010)

- $J > 1$ crucial to replicate the boom-bust cycle after a monetary shock

Outline for discussion

1. contribution of the paper
2. when does DE on endogenous variables matter?
3. where does the reversal/ boom-bust cycle come from?

Diagnostic Expectations

- Consider the process

$$x_t = \rho_x x_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$

- Diagnostic pdf is defined as

$$\log f_t^\theta(x_{t+1}) = \underbrace{\log f(x_{t+1}|G_t)}_{\text{RE}} + \underbrace{\theta \left(\log f(x_{t+1}|G_t) - \log f(x_{t+1}|G_t^r)\right)}_{\text{distortion}} + C, \quad \theta > 0$$

- Information sets:
  - G_t: current state t
  - G_t^r: reference state. (BIS: $t - J$, with $J > 1$)
    (Follow Bordalo, Gennaioli & Shleifer (2018))

  θ: degree of diagnosticity

Formula for Univariate Case and Example
when $J = 1$

- Diagnostic expectation is:

$$\mathbb{E}_t^{\theta,1}[x_{t+1}] = \mathbb{E}_t[x_{t+1}] + \underbrace{\theta(\mathbb{E}_t[x_{t+1}] - \mathbb{E}_{t-1}[x_{t+1}])}_{\text{distortion}}$$

- We have that:

$\mathbb{E}_t[x_{t+1}] = \rho_x \check{x}_t$ and $\mathbb{E}_{t-1}[x_{t+1}] = \rho_x^2 \check{x}_{t-1}$

- So:

$$\mathbb{E}_t^{\theta,1}[x_{t+1}] = \rho_x \check{x}_t + \theta(\rho_x \check{x}_t - \rho_x^2 \check{x}_{t-1}) = \rho_x \check{x}_t + \theta \rho_x \check{\varepsilon}_t$$

$\implies$ extrapolation or over-reaction

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$\Longrightarrow$ extrapolation or over-reaction

Empirical support for extrapolation : Greenwood-Shleifer (2014), Bordalo-La Porta-Gennaioli-Shleifer (2019), Bordalo-Gennaioli-Ma-Shleifer (2020) , Broer-Kohlhas (2020), Angeletos-Huo-Sastry (2021), Kohlhas-Walther (2021),...

DE when reference period is in *distant past*

$$\mathbb{E}_t^{\theta,J}[x_{t+1}] = \mathbb{E}_t[x_{t+1}] + \theta \underbrace{\left(\mathbb{E}_t[x_{t+1}] - \sum_{j=1}^{J} \alpha_{j,J}\mathbb{E}_{t-j}[x_{t+1}]\right)}_{\text{weighted average of forecast revisions}}; \quad \sum_{j=1}^{J} \alpha_{j,J} = 1$$

For $J = 1, 2, 3, \dots$

$$\mathbb{E}_t^{\theta,1}[x_{t+1}] = \mathbb{E}_t[x_{t+1}] + \theta \left(\mathbb{E}_t[x_{t+1}] - \mathbb{E}_{t-1}[x_{t+1}]\right)$$

$$\mathbb{E}_t^{\theta,2}[x_{t+1}] = \mathbb{E}_t[x_{t+1}] + \theta \left(\mathbb{E}_t[x_{t+1}] - \sum_{j=1}^{2} \alpha_{j,2}\mathbb{E}_{t-j}[x_{t+1}]\right)$$

$$\mathbb{E}_t^{\theta,3}[x_{t+1}] = \mathbb{E}_t[x_{t+1}] + \theta \left(\mathbb{E}_t[x_{t+1}] - \sum_{j=1}^{3} \alpha_{j,3}\, \mathbb{E}_{t-j}[x_{t+1}]\right)$$

$\theta = 0$ corresponds to Rational Expectations (RE)

Diagnostic Expectations in macro models

Bordalo, Gennaioli, Shleifer & Terry (2021)

- financial frictions interact with DE in a heterogenous firm RBC model

Maxted (2020), Farhi & Werning (2021), Krishnamurthy and Li (2021),...

- DE can help construct predictable financial crises, macro pru implications

Bianchi, Ilut & Saijo (2021) and L'Huillier, Singh & Yoo (2021)

- incorporate DE on endogenous variables in linear GE

(many other references on the use of extrapolative expectations in macro-finance)

When does DE on endogenous variables matter?

$$y_t = a\,\tilde{\mathbb{E}}_t y_{t+1} + c\,y_{t-1} + \epsilon_t; \quad \epsilon_t \sim iid\,N(0,1)$$

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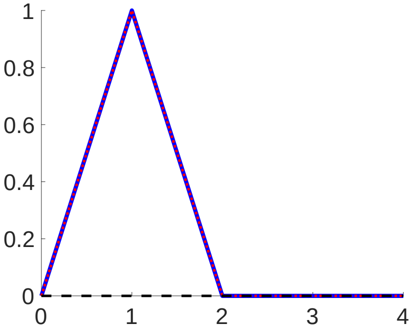
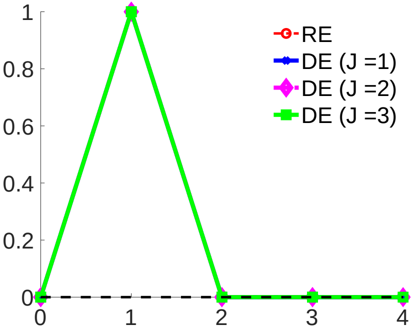
Assume $a = 0.5$, $c = 0$



Shock process ϵ_t



Solution for y_t when $c = 0$

When does DE on endogenous variables matter?

Solution with $J = 1$ ($a = 0.5$, $c = 0.4$, $\theta = 1$)

$$y_t = a\,\mathbb{E}_t^{\theta,1} y_{t+1} + c\,y_{t-1} + \epsilon_t; \quad \epsilon_t \sim iid\,N(0,1)$$

$$\mathbb{E}_t^{\theta,1} y_{t+1} = (1+\theta)\mathbb{E}_t y_{t+1} - \theta \mathbb{E}_{t-1} y_{t+1}$$

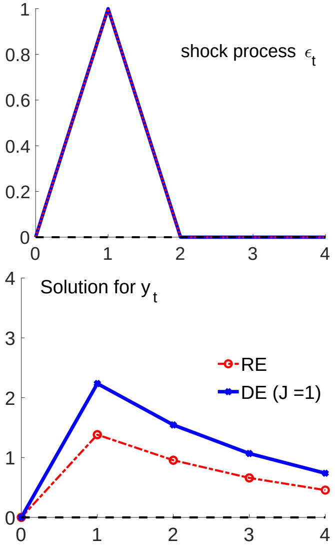
RE ($\theta = 0$):

$$y_t = \phi y_{t-1} + \frac{1}{1 - a\phi}\epsilon_1$$

DE at $J = 1$:

$$y_t = \phi y_{t-1} + \frac{1}{1 - (1+\theta)a\phi}\epsilon_1$$

where $\phi \equiv \frac{1-\sqrt{1-4ac}}{2a}$



Where does boom-bust cycle come from?

- Consider

$$\frac{u'(C_t)}{P_t} = \beta(1+i_t)\mathbb{E}_t^{\theta,J}\left[\frac{u'(C_{t+1})}{P_{t+1}}\right]$$

- Notice,

$$\mathbb{E}_t^{\theta,J}[X_{t+1}Y_t] \neq \mathbb{E}_t^{\theta,J}[X_{t+1}]Y_t$$

- When $J=1$, use conditioning on $t-1$:

$$u'(C_t)\frac{P_{t-1}}{P_t} = \beta(1+i_t)\mathbb{E}_t^{\theta,1}\left[u'(C_{t+1})\frac{P_{t-1}}{P_t}\frac{P_t}{P_{t+1}}\right]$$

  and approximate

Obtaining Diagnostic Fisher Equation

- We have:

$$u'(C_t)\frac{P_{t-1}}{P_t} = \beta(1+i_t)\mathbb{E}_t^{\theta,1}\left[u'(C_{t+1})\frac{P_{t-1}}{P_t}\frac{P_t}{P_{t+1}}\right]$$

- Resulting diagnostic Fisher equation ($J = 1$):

$$\hat{r}_t = \hat{i}_t - \mathbb{E}_t[\pi_{t+1}] - \underbrace{\theta(\mathbb{E}_t[\pi_{t+1}] - \mathbb{E}_{t-1}[\pi_{t+1}])}_{\frac{P_t}{P_{t+1}}} - \underbrace{\theta(\pi_t - \mathbb{E}_{t-1}[\pi_t])}_{\frac{P_{t-1}}{P_t}\text{(momentum)}}$$

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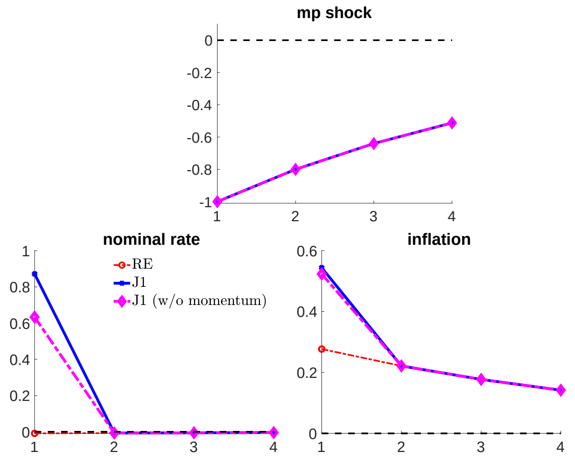
100 bps $\downarrow \epsilon_t^{mp}$ & $J = 1$

Taylor Rule: $\hat{i}_t = 1.50\pi_t + 0.5\hat{x}_t + m_t; m_t = 0.8m_{t-1} + \epsilon_t^{mp}$



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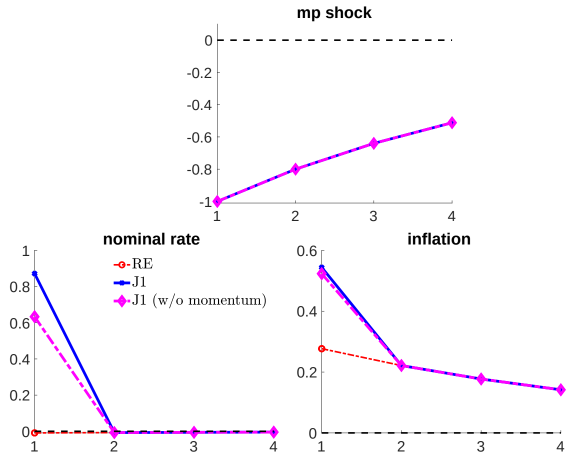

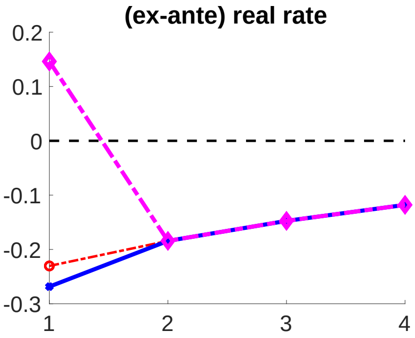

$$\hat{r}_t = \hat{i}_t - \mathbb{E}_t^{\theta,1}[\pi_{t+1}] - \underbrace{\theta(\pi_t - \mathbb{E}_{t-1}[\pi_t])}_{\frac{P_{t-1}}{P_t}\text{(momentum)}}$$

NK model calibration: Galí (2015) textbook ($\beta = 0.99$, $\kappa = 0.05$) + DE parameter ($\theta = 1$). See: L'Huillier, Singh and Yoo (2021)

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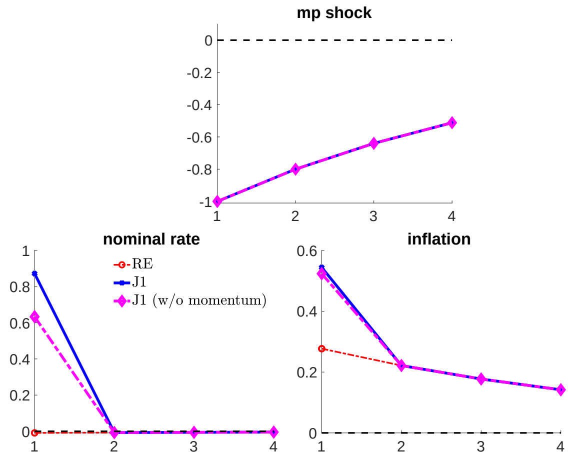

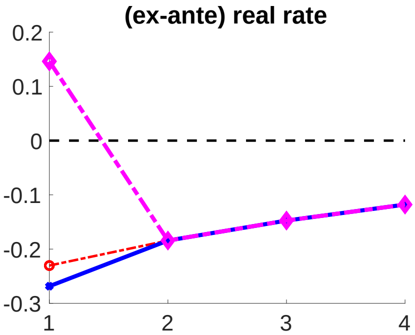

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Similar mechanism gives rise to high fiscal multiplier

NK model calibration: Galí (2015) textbook ($\beta = 0.99$, $\kappa = 0.05$) + DE parameter ($\theta = 1$). See: L'Huillier, Singh and Yoo (2021)

Diagnostic Fisher Equation when $J > 1$

$$\hat{r}_t = \hat{i}_t - \mathbb{E}_t[\pi_{t+1}] - \underbrace{\theta \left(\mathbb{E}_t[\pi_{t+1}] - \sum_{j=1}^{J} \alpha_{j,J} \mathbb{E}_{t-j}[\pi_{t+1}] \right)}_{\frac{P_t}{P_{t+1}}} - \underbrace{\theta \sum_{j=0}^{J-1} \left(\pi_{t-j} - \mathbb{E}^r_{t-1}[\pi_{t-j}] \right)}_{\frac{P_{t-J}}{P_t} \text{(momentum)}}$$

$\mathbb{E}^r_{t-1}[\pi_t] = \sum_{k=1}^{J} \alpha_{k,J} \mathbb{E}_{t-k}[\pi_t]$ is expectation of current inflation formed during reference periods in the distant past.

Diagnostic Fisher Equation when $J > 1$

$$\hat{r}_t = \hat{i}_t - \mathbb{E}_t^{\theta,J}[\pi_{t+1}] \qquad \underbrace{-\theta \sum_{j=0}^{J-1} \left(\pi_{t-j} - \mathbb{E}_{t-1}^{r}[\pi_{t-j}]\right)}_{\frac{P_{t-J}}{P_t}\text{(momentum)}}$$

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Past inflation surprises accumulate in agent's memory

$\rightarrow$ make future price level seem *very* high

Crucial mechanism for their estimated DSGE model.

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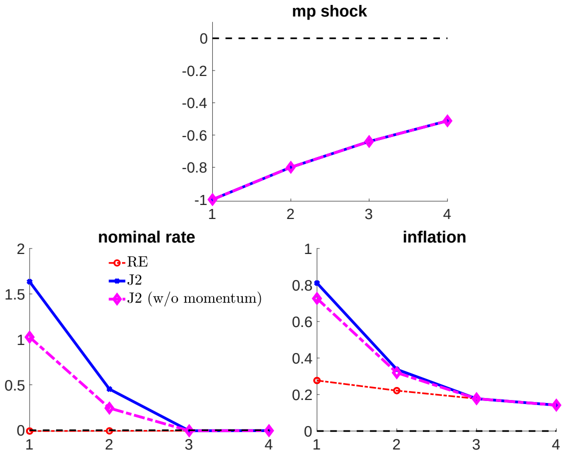

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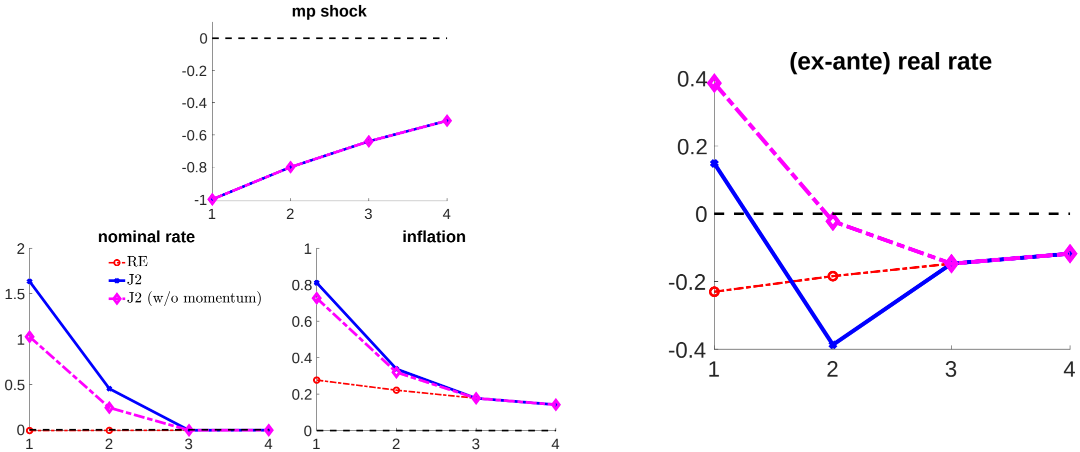
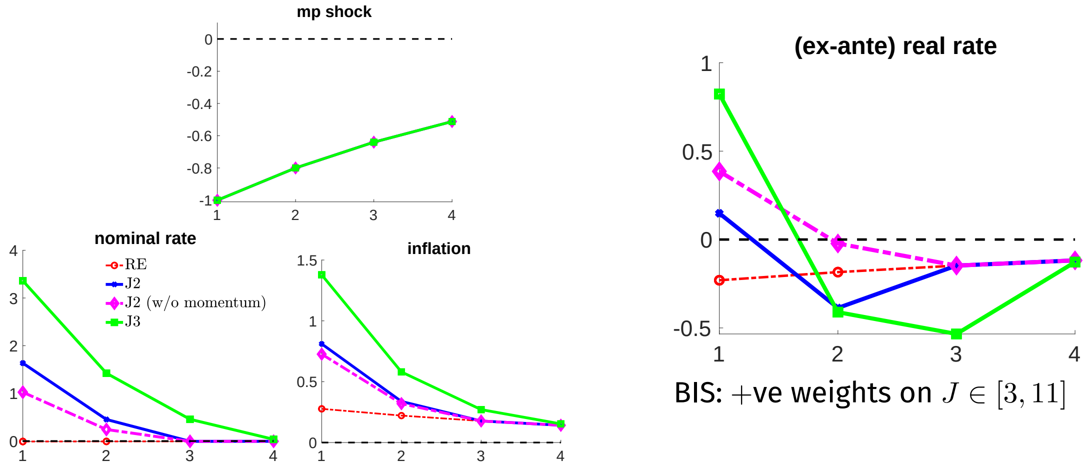


NK model calibration: Galí (2015) textbook ($\beta = 0.99$, $\kappa = 0.05$) + DE parameter ($\theta = 1$). Equal weights on $J = 1$ and $J = 2$ reference periods

100 bps $\downarrow \epsilon_t^{mp}$ & $J = 3$

Taylor Rule: $\hat{i}_t = 1.50\pi_t + 0.5\hat{x}_t + m_t; m_t = 0.8m_{t-1} + \epsilon_t^{mp}$



BIS: $+$ve weights on $J \in [3, 11]$

NK model calibration: Galí (2015) textbook ($\beta = 0.99$, $\kappa = 0.05$) + DE parameter ($\theta = 1$). Equal weights on $J = 1$, $J = 2$ and $J = 3$ reference

Summary

- how to integrate diagnostic expectations into linear models
- authors break a lot of ground in this territory with careful analysis
- many more goods in the paper

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Thank You!

Appendix

Solution with $J = 2$ ($a = 0.5$, $c = 0.4$, $\theta = 1$)

$$y_t = a\,\mathbb{E}_t^{\theta,2} y_{t+1} + c\,y_{t-1} + \epsilon_t; \quad \epsilon_t \sim iid\,N(0,1)$$

$$\mathbb{E}_t^{\theta,2} y_{t+1} = (1+\theta)\mathbb{E}_t[y_{t+1}] - \frac{\theta}{2}\sum_{j=1}^{2}\mathbb{E}_{t-j}[y_{t+1}]$$

Solution for DE at $J = 2$:

$$y_1 = \frac{1 - a\phi(1 + 0.5\,\theta)}{1 - a\phi(1 + 0.5\,\theta) - ac(1+\theta)}\epsilon_1 > \underbrace{\frac{1}{1 - a\phi}\epsilon_1}_{y_1^{RE}}$$

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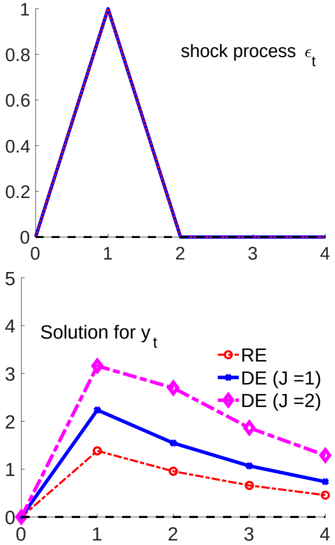
$$y_2 = \frac{c}{1 - a\phi(1 + 0.5\,\theta)} y_1 > \phi y_1$$

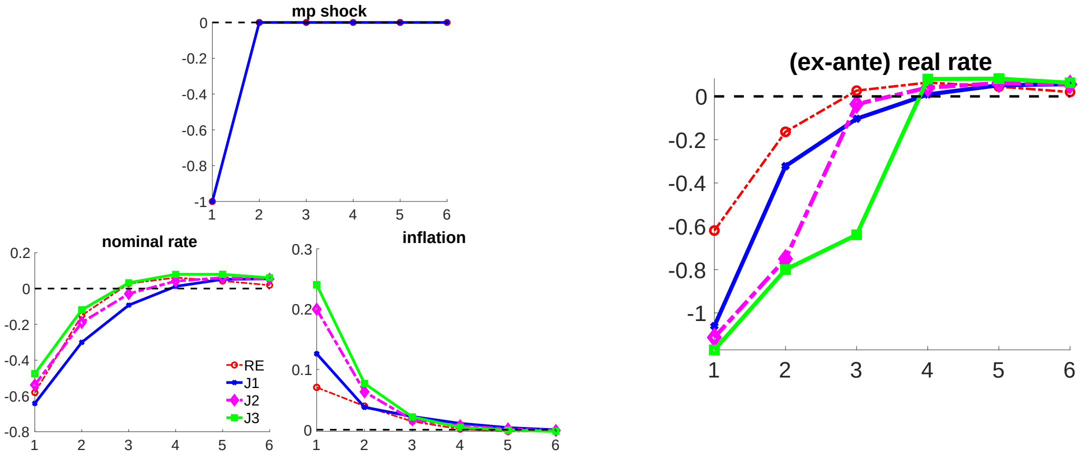
$$y_3 = \phi y_2$$

where $\phi \equiv \frac{1-\sqrt{1-4ac}}{2a}$



100 bps $\downarrow \epsilon_t^{mp}$ & $J = 3$ (consumption habits + policy inertia)

Taylor Rule: $\hat{i}_t = 0.8\hat{i}_{t-1} + 1.50\pi_t + 0.5\hat{x}_t + \epsilon_t^{mp}$; external consumption habits in utility: $\log\left(C_t - 0.6\bar{C}_{t-1}\right)$



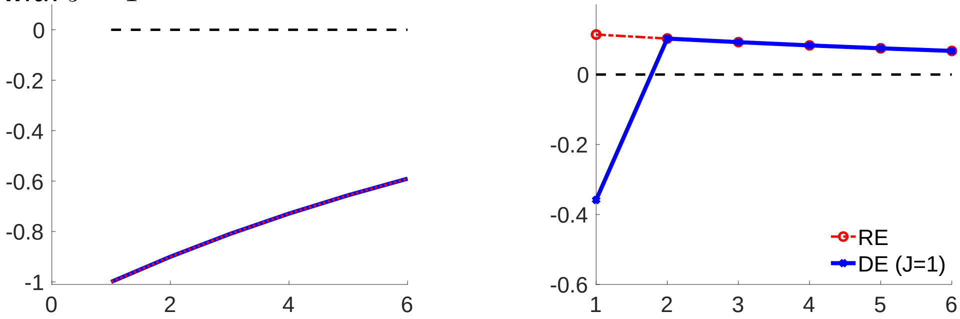
Three equation NK model calibration: Galí (2015) textbook ($\beta = 0.99$, $\kappa = 0.05$), habit ($h = 0.6$) + DE parameter ($\theta = 1$).

Unit weight on the most distant reference date $\alpha_J = 1 \; \forall J \in \{1, 2, 3\}$.

Reversal to Rationality: Output Gap after negative TFP shock

Taylor Rule: $\hat{i}_t = 1.50\pi_t + 0.5\hat{x}_t$; TFP process $a_t = 0.9a_{t-1} + \epsilon_t$

DE with $J = 1$



TFP shock process

Output Gap

Intuition: DE agent expects TFP to fall by a lot, (in excess of reality)

$\Longrightarrow$ Persistent drop in consumption

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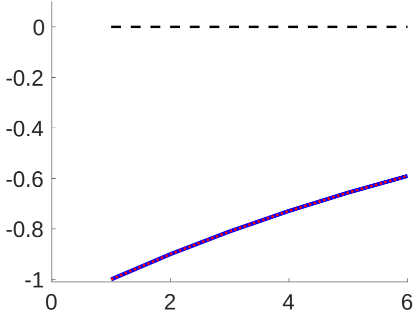

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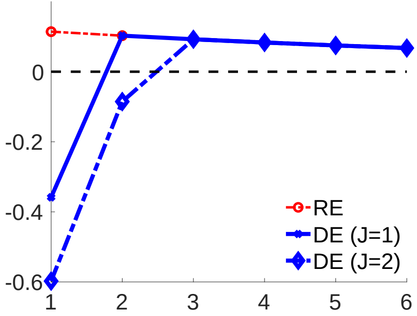

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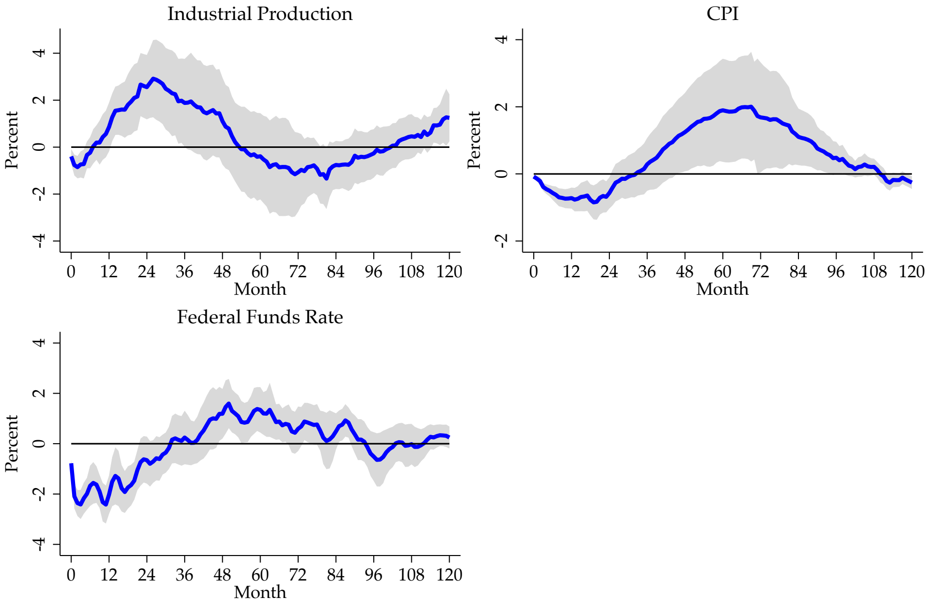
Empirical IRFs to Romer and Romer (2004) shocks: monthly frequency

$$x_{t+h} = c_h + \tau_h\, t + \beta_h^M \varepsilon_t^{RR} + \Gamma Z_t + \eta_{t+h}; \quad h = 0, ..., H$$

- Jordá (2005) Local Projections
- β_h^M directly plots the causal effect of monetary shock on x at horizon h
- Monthly data from FRED (INDPRO, CPIAUCSL, FEDFUNDS)
- Romer and Romer (2004) shocks from Wieland and Yang (2019)
- Z_t includes 12 (48) monthly lags of the LHS (x_{t-i}) and the RR shocks variable
- t: is a linear time-trend
- Shaded areas denote 95% robust HAC standard error bands

Empirical IRFs to Romer and Romer (2004) shocks: monthly frequency

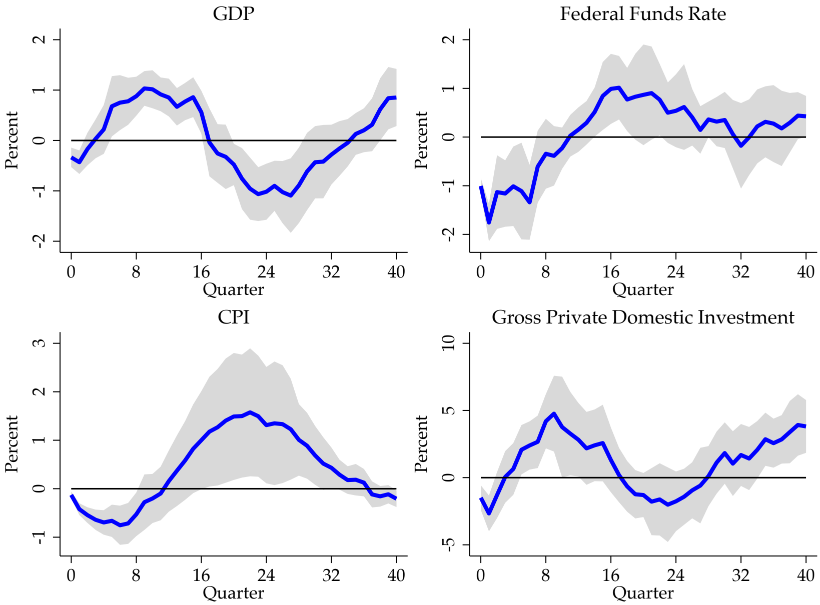


Empirical IRFs to Romer and Romer (2004) shocks: quarterly frequency

$$x_{t+h} = c_h + \tau_h\, t + \beta_h^Q \varepsilon_t^{RR} + \Gamma Z_t + \eta_{t+h}; \quad h = 0, ..., H$$

- Jordá (2005) Local Projections
- β_h^Q directly plots the causal effect of monetary shock on x at horizon h
- Quarterly data from FRED (GDPC1, CPIAUCSL, FEDFUNDS, GPDIC1, B230RC0Q173SBEA, PCDG/DDURRD3Q086SBEA) + McKay and Wieland (2021) replication package (qres, qall, qnonall)
- Romer and Romer (2004) quarterly shocks from Wieland and Yang (2019)
- Z_t includes 16 quarterly lags of the LHS (x_{t-i}) and RR shocks
- t: is a linear time-trend
- Shaded areas denote 95% robust HAC standard error bands

Empirical IRFs to Romer and Romer (2004) shocks: quarterly frequency



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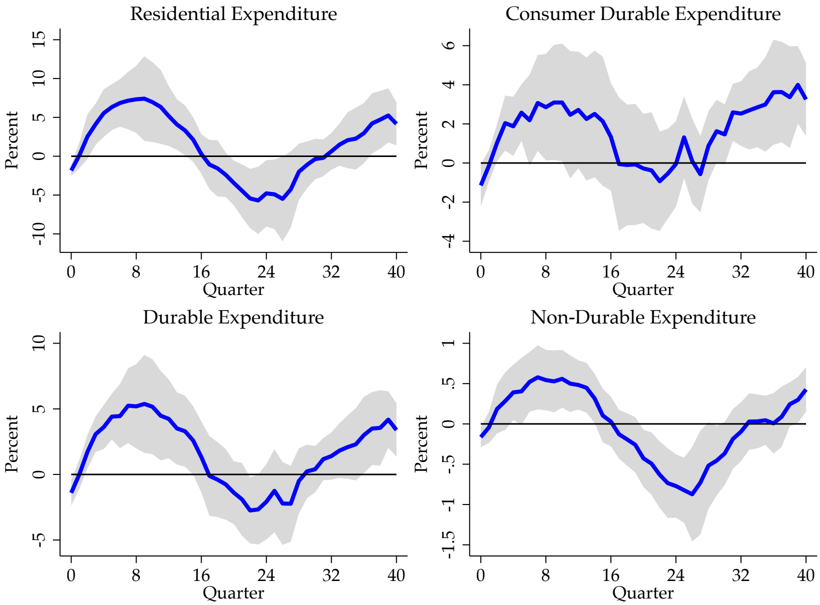

Additional Comments

- Beaudry-Portier /McKay-Wieland stories require a non-linear model for boom-bust. This is a linear model!
- DSGE model matches the survey expectations IRF. Very nice!
- Monthly IRFs noisy: is there a frequency implication in the model?
- Boom-bust story seems to be largely residential and non-durable expenditure?
- What is the model fit of RE vs DE based only on 20 quarters IRF?
- Are IRFs beyond horizon 20, from 120 quarters data, reliable?
  Jordá, Singh & Taylor (2021): The Long-run Effects of Monetary Policy,

  Bernanke & Mihov (1998): The Liquidity Effect and Long-Run Neutrality
- DE $\approx$ endogenous news shocks (compare with exogenous news/noise shocks)?