Discussion of
Diagnostic Business Cycles
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What this paper is about?

- develop tools to incorporate Diagnostic Expectations (DE) in linear GE economies
  1. endogenous states $\rightarrow$ endogenous amplification
  2. selective memory recall based on distant past $\rightarrow$ boom/bust cycle

- estimate a medium-scale DSGE model with DE
  (Christiano Eichenbaum Evans 2005, Christiano Trabandt Walentin 2010)

- $J > 1$ crucial to replicate the boom-bust cycle after a monetary shock
Outline for discussion

1. contribution of the paper
2. when does DE on endogenous variables matter?
3. where does the reversal/boom-bust cycle come from?
Diagnostic Expectations

- Consider the process

\[ x_t = \rho x_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma^2_\varepsilon) \]

- Diagnostic pdf is defined as

\[
\log f_t^\theta (x_{t+1}) = \log f(x_{t+1} | G_t) + \theta (\log f(x_{t+1} | G_t) - \log f(x_{t+1} | G_{t}^r)) + C, \quad \theta > 0
\]

- Information sets:
  - \( G_t \): current state \( t \)
  - \( G_t^r \): reference state. (BIS: \( t - J \), with \( J > 1 \))
    (Follow Bordalo, Gennaioli & Shleifer (2018))

\( \theta \): degree of diagnosticity
Formula for Univariate Case and Example
when $J = 1$

- **Diagnostic expectation is:**

  $E^{\theta,1}_{t}[x_{t+1}] = E_t[x_{t+1}] + \theta(E_t[x_{t+1}] - E_{t-1}[x_{t+1}])$

- **We have that:**

  $E_t[x_{t+1}] = \rho_x \tilde{x}_t \text{ and } E_{t-1}[x_{t+1}] = \rho_x^2 \tilde{x}_{t-1}$

- **So:**

  $E^{\theta,1}_{t}[x_{t+1}] = \rho_x \tilde{x}_t + \theta(\rho_x \tilde{x}_t - \rho_x^2 \tilde{x}_{t-1}) = \rho_x \tilde{x}_t + \theta \rho_x \tilde{\epsilon}_t$

  $\implies$ **extrapolation or over-reaction**

Empirical support for extrapolation: Greenwood-Shleifer, Bordalo-La Porta-Gennaioli-Shleifer, Bordalo-Gennaioli-Ma-Shleifer, Broer-Kohlhas, Angeletos-Huo-Sastry, Kohlhas-Walther, ...
Formula for Univariate Case and Example when $J = 1$

- Diagnostic expectation is:
  \[
  E_t^{\theta,1}[x_{t+1}] = E_t[x_{t+1}] + \theta(E_t[x_{t+1}] - E_{t-1}[x_{t+1}])
  \]

- We have that:
  \[
  E_t[x_{t+1}] = \rho_x \tilde{x}_t \quad \text{and} \quad E_{t-1}[x_{t+1}] = \rho_x^2 \tilde{x}_{t-1}
  \]

- So:
  \[
  E_t^{\theta,1}[x_{t+1}] = \rho_x \tilde{x}_t + \theta(\rho_x \tilde{x}_t - \rho_x^2 \tilde{x}_{t-1}) = \rho_x \tilde{x}_t + \theta \rho_x \tilde{\epsilon}_t
  \]

\[\implies\] extrapolation or over-reaction

DE when reference period is in *distant past*

\[
\mathbb{E}_t^{\theta,J}[x_{t+1}] = \mathbb{E}_t[x_{t+1}] + \theta \left( \mathbb{E}_t[x_{t+1}] - \sum_{j=1}^{J} \alpha_{j,J} \mathbb{E}_{t-j}[x_{t+1}] \right); \quad \sum_{j=1}^{J} \alpha_{j,J} = 1
\]

weighted average of forecast revisions

For \( J = 1, 2, 3, \ldots \)

\[
\mathbb{E}_t^{\theta,1}[x_{t+1}] = \mathbb{E}_t[x_{t+1}] + \theta \left( \mathbb{E}_t[x_{t+1}] - \mathbb{E}_{t-1}[x_{t+1}] \right)
\]

\[
\mathbb{E}_t^{\theta,2}[x_{t+1}] = \mathbb{E}_t[x_{t+1}] + \theta \left( \mathbb{E}_t[x_{t+1}] - \sum_{j=1}^{2} \alpha_{j,2} \mathbb{E}_{t-j}[x_{t+1}] \right)
\]

\[
\mathbb{E}_t^{\theta,3}[x_{t+1}] = \mathbb{E}_t[x_{t+1}] + \theta \left( \mathbb{E}_t[x_{t+1}] - \sum_{j=1}^{3} \alpha_{j,3} \mathbb{E}_{t-j}[x_{t+1}] \right)
\]

\( \theta = 0 \) corresponds to Rational Expectations (RE)
Diagnostic Expectations in macro models

Bordalo, Gennaioli, Shleifer & Terry (2021)
► financial frictions interact with DE in a heterogenous firm RBC model

Maxted (2020), Farhi & Werning (2021), Krishnamurthy and Li (2021),...
► DE can help construct predictable financial crises, macro pru implications

Bianchi, Ilut & Saijo (2021) and L’Huillier, Singh & Yoo (2021)
► incorporate DE on endogenous variables in linear GE

(many other references on the use of extrapolative expectations in macro-finance)
When does DE on endogenous variables matter?

\[ y_t = a \tilde{E}_t y_{t+1} + c y_{t-1} + \epsilon_t; \quad \epsilon_t \sim iid N(0, 1) \]
When does DE on endogenous variables matter?

\[ y_t = a \tilde{E}_t y_{t+1} + c y_{t-1} + \epsilon_t; \quad \epsilon_t \sim iid N(0, 1) \]

Assume \( a = 0.5, c = 0 \)

Shock process \( \epsilon_t \)

Solution for \( y_t \) when \( c = 0 \)
When does DE on endogenous variables matter?

Solution with $J = 1$ ($a = 0.5$, $c = 0.4$, $\theta = 1$)

$$y_t = a \mathbb{E}_t^{1} y_{t+1} + c y_{t-1} + \epsilon_t; \quad \epsilon_t \sim iid \mathcal{N}(0, 1)$$

$$\mathbb{E}_t^{1} y_{t+1} = (1 + \theta) \mathbb{E}_t y_{t+1} - \theta \mathbb{E}_{t-1} y_{t+1}$$

RE ($\theta = 0$):

$$y_t = \phi y_{t-1} + \frac{1}{1 - a\phi} \epsilon_1$$

DE at $J = 1$:

$$y_t = \phi y_{t-1} + \frac{1}{1 - (1 + \theta)a\phi} \epsilon_1$$

where $\phi \equiv \frac{1 - \sqrt{1 - 4ac}}{2a}$
Where does boom-bust cycle come from?

Consider

\[
\frac{u'(C_t)}{P_t} = \beta(1 + i_t) E_{\theta,J}^{\theta,J} \left[ \frac{u'(C_{t+1})}{P_{t+1}} \right]
\]

Notice,

\[
E_{\theta,J}^{\theta,J} [X_{t+1} Y_t] \neq E_{\theta,J}^{\theta,J} [X_{t+1}] Y_t
\]

When \( J = 1 \), use conditioning on \( t - 1 \):

\[
\frac{u'(C_t) P_{t-1}}{P_t} = \beta(1 + i_t) E_{\theta,1}^{\theta,1} \left[ \frac{u'(C_{t+1})}{P_{t+1}} \right]
\]

and approximate
Obtaining Diagnostic Fisher Equation

► We have:

\[ u'(C_t) \frac{P_{t-1}}{P_t} = \beta (1 + i_t) E_t^{\theta,1} \left[ u'(C_{t+1}) \frac{P_{t-1}}{P_t} \frac{P_t}{P_{t+1}} \right] \]

► Resulting diagnostic Fisher equation \((J = 1)\):

\[ \hat{r}_t = \hat{i}_t - E_t[\pi_{t+1}] - \theta (E_t[\pi_{t+1}] - E_{t-1}[\pi_{t+1}]) - \theta (\pi_t - E_{t-1}[\pi_t]) \]

\[ \frac{P_t}{P_{t+1}} \] (momentum)
Obtaining Diagnostic Fisher Equation

We have:

\[ u'(C_t) \frac{P_{t-1}}{P_t} = \beta (1 + i_t) \mathbb{E}^{\theta,1}_t \left[ u'(C_{t+1}) \frac{P_{t-1}}{P_t} \frac{P_t}{P_{t+1}} \right] \]

Resulting diagnostic Fisher equation \((J = 1)\):

\[ \hat{r}_t = \hat{i}_t - \mathbb{E}_t[\pi_{t+1}] - \theta (\mathbb{E}_t[\pi_{t+1}] - \mathbb{E}_{t-1}[\pi_{t+1}]) - \theta (\pi_t - \mathbb{E}_{t-1}[\pi_t]) \left( \mathbb{E}^{\theta,1}_t[\pi_{t+1}] \right) \left( \frac{P_{t-1}}{P_t} \right) \text{ (momentum)} \]
Obtaining Diagnostic Fisher Equation

We have:

\[ u'(C_t) \frac{P_{t-1}}{P_t} = \beta (1 + i_t) \mathbb{E}_t^{\theta,1} \left[ u'(C_{t+1}) \frac{P_{t-1}}{P_t} \frac{P_t}{P_{t+1}} \right] \]

Resulting Diagnostic Fisher equation \((J = 1)\):

\[ \hat{r}_t = \hat{i}_t - \mathbb{E}_t^{\theta,1}[\pi_{t+1}] - \theta (\pi_t - \mathbb{E}_{t-1}[\pi_t]) \]

\[
\frac{P_{t-1}}{P_t} \text{(momentum)}
\]
$100 \text{ bps} \downarrow \epsilon_t^{mp} \text{ & } J = 1$

Taylor Rule: $\hat{i}_t = 1.50 \pi_t + 0.5 \bar{x}_t + m_t; m_t = 0.8 m_{t-1} + \epsilon_t^{mp}$
100 bps $\epsilon_t^{mp}$ & $J = 1$

Taylor Rule: $\hat{i}_t = 1.50\pi_t + 0.5\tilde{x}_t + m_t; m_t = 0.8m_{t-1} + \epsilon_t^{mp}$

\[ \hat{r}_t = \hat{i}_t - E_t^{\theta,1} [\pi_{t+1}] - \theta (\pi_t - E_{t-1}[\pi_t]) \]

NK model calibration: Gali (2015) textbook ($\beta = 0.99, \kappa = 0.05$) + DE parameter ($\theta = 1$). See: L'Huillier, Singh and Yoo (2021)
100 bps $\downarrow \epsilon_{tm}^m \& J = 1$

Taylor Rule: $\hat{i}_t = 1.50\pi_t + 0.5\hat{x}_t + m_t; m_t = 0.8m_{t-1} + \epsilon_{tm}$
Diagnostic Fisher Equation when $J > 1$

$$\hat{r}_t = \hat{\pi}_t - \mathbb{E}_t[\pi_{t+1}] - \theta \left( \mathbb{E}_t[\pi_{t+1}] - \sum_{j=1}^{J} \alpha_{j,J} \mathbb{E}_{t-j}[\pi_{t+1}] \right) - \theta \sum_{j=0}^{J-1} \left( \pi_{t-j} - \mathbb{E}_{t-1}^r[\pi_{t-j}] \right)$$

$$\frac{P_t}{P_{t+1}}$$

$\mathbb{E}_{t-1}^r[\pi_t] = \sum_{k=1}^{J} \alpha_{k,J} \mathbb{E}_{t-k}[\pi_t]$ is expectation of current inflation formed during reference periods in the distant past.
Diagnostic Fisher Equation when $J > 1$

\[
\hat{r}_t = \hat{\iota}_t - \mathbb{E}_t^\Theta,J [\pi_{t+1}]
\]

\[
-\theta \sum_{j=0}^{J-1} \left( \pi_{t-j} - \mathbb{E}_{t-1}^r [\pi_{t-j}] \right)
\]

\[
\frac{P_{t-J}}{P_t} \text{(momentum)}
\]

$\mathbb{E}_{t-1}^r [\pi_t] = \sum_{k=1}^{J} \alpha_k J \mathbb{E}_{t-k} [\pi_t]$ is expectation of current inflation formed during reference periods in the distant past.
Diagnostic Fisher Equation when $J > 1$

$$\hat{r}_t = \hat{i}_t - \mathbb{E}^{\theta,J}_t [\pi_{t+1}] - \theta \sum_{j=0}^{J-1} (\pi_{t-j} - \mathbb{E}^{\theta}_{t-1} \pi_{t-j})$$

$$\frac{P_t - J}{P_t} (\text{momentum})$$

$$\mathbb{E}^{\theta}_{t-1} [\pi_t] = \sum_{k=1}^{J} \alpha_{k,J} \mathbb{E}_{t-k} [\pi_t]$$ is expectation of current inflation formed during reference periods in the distant past.
Diagnostic Fisher Equation when $J > 1$

\[
\hat{r}_t = \hat{i}_t - \mathbb{E}_t^{\theta,J}[\pi_{t+1}] - \theta \sum_{j=0}^{J-1} \left( \pi_{t-j} - \mathbb{E}_{t-1}^{T}[\pi_{t-j}] \right)
\]

\[
\frac{P_{t-J}}{P_t} \text{(momentum)}
\]

Past inflation surprises accumulate in agent’s memory
→ make future price level seem very high

Crucial mechanism for their estimated DSGE model.

\[
\mathbb{E}_{t-1}^{T}[\pi_t] = \sum_{k=1}^{J} \alpha_k \mathbb{E}_{t-k}[\pi_t] \text{ is expectation of current inflation formed during reference periods in the distant past.}
\]
100 bps ↓ $\epsilon_t^{mp}$ & $J = 2$

Taylor Rule: \( \hat{i}_t = 1.50\pi_t + 0.5\bar{x}_t + m_t; m_t = 0.8m_{t-1} + \epsilon_t^{mp} \)
100 bps \downarrow \epsilon_t^{mp} \& J = 2

Taylor Rule: \hat{i}_t = 1.50\pi_t + 0.5\bar{x}_t + m_t; m_t = 0.8m_{t-1} + \epsilon_t^{mp}

NK model calibration: Gali (2015) textbook (\beta = 0.99, \kappa = 0.05) \& DE parameter (\theta = 1). Equal weights on \( J = 1 \) and \( J = 2 \) reference periods
$\text{100~bps} \downarrow \epsilon_t^{mp} \& J = 3$

Taylor Rule: $\hat{i}_t = 1.50\pi_t + 0.5\bar{x}_t + m_t; m_t = 0.8m_{t-1} + \epsilon_t^{mp}$
Summary

- how to integrate diagnostic expectations into linear models
- authors break a lot of ground in this territory with careful analysis
- many more goods in the paper
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- how to integrate diagnostic expectations into linear models
- authors break a lot of ground in this territory with careful analysis
- many more goods in the paper

Thank You!
Appendix
Solution with $J = 2$ ($a = 0.5$, $c = 0.4$, $\theta = 1$)

$$y_t = a \mathbb{E}_t^{\theta; 2} y_{t+1} + c y_{t-1} + \epsilon_t; \quad \epsilon_t \sim iid N(0, 1)$$

$$\mathbb{E}_t^{\theta; 2} y_{t+1} = (1+\theta) \mathbb{E}_t [y_{t+1}] - \frac{\theta}{2} \sum_{j=1}^{2} \mathbb{E}_{t-j}[y_{t+1}]$$

Solution for DE at $J = 2$:

$$y_1 = \frac{1 - a \phi (1 + 0.5 \theta)}{1 - a \phi (1 + 0.5 \theta) - ac(1 + \theta)} \epsilon_1 > \frac{1}{1 - a \phi} \epsilon_1$$

where $\phi \equiv \frac{1 - \sqrt{1 - 4ac^2}}{2a}$.
Solution with $J = 2$ ($a = 0.5, c = 0.4, \theta = 1$)

\[ y_t = a \mathbb{E}_t^{\theta^2} y_{t+1} + c y_{t-1} + \epsilon_t; \quad \epsilon_t \sim iid \mathcal{N}(0, 1) \]

\[ \mathbb{E}_t^{\theta^2} y_{t+1} = (1+\theta)\mathbb{E}_t[y_{t+1}] - \frac{\theta}{2} \sum_{j=1}^{2} \mathbb{E}_{t-j}[y_{t+1}] \]

Solution for DE at $J = 2$:

\[ y_1 = \frac{1 - a\phi(1 + 0.5 \theta)}{1 - a\phi(1 + 0.5 \theta) - ac(1 + \theta)} \epsilon_1 > \frac{1}{1 - a\phi} \epsilon_1 \]

\[ y_2 = \frac{c}{1 - a\phi(1 + 0.5 \theta)} y_1 > \phi y_1 \]

\[ y_3 = \phi y_2 \]

where $\phi \equiv \frac{1 - \sqrt{1 - 4ac}}{2a}$

\[ y_t = a \mathbb{E}_t^{\theta^2} y_{t+1} + c y_{t-1} + \epsilon_t; \quad \epsilon_t \sim iid \mathcal{N}(0, 1) \]

\[ \mathbb{E}_t^{\theta^2} y_{t+1} = (1+\theta)\mathbb{E}_t[y_{t+1}] - \frac{\theta}{2} \sum_{j=1}^{2} \mathbb{E}_{t-j}[y_{t+1}] \]

Solution for DE at $J = 2$:

\[ y_1 = \frac{1 - a\phi(1 + 0.5 \theta)}{1 - a\phi(1 + 0.5 \theta) - ac(1 + \theta)} \epsilon_1 > \frac{1}{1 - a\phi} \epsilon_1 \]

\[ y_2 = \frac{c}{1 - a\phi(1 + 0.5 \theta)} y_1 > \phi y_1 \]

\[ y_3 = \phi y_2 \]

where $\phi \equiv \frac{1 - \sqrt{1 - 4ac}}{2a}$
100 bps ↓ $\epsilon_{t}^{mp}$ & $J = 3$ (consumption habits + policy inertia)

Taylor Rule: $\hat{i}_{t} = 0.8\hat{i}_{t-1} + 1.50\pi_{t} + 0.5\bar{x}_{t} + \epsilon_{t}^{mp};$ external consumption habits in utility: $\log (C_{t} - 0.6\bar{C}_{t-1})$
Reversal to Rationality: Output Gap after negative TFP shock

Taylor Rule: \( \hat{i}_t = 1.50\pi_t + 0.5\hat{x}_t \); TFP process \( a_t = 0.9a_{t-1} + \epsilon_t \)

\[ \text{DE with } J = 1 \]

![Graph showing TFP shock process and Output Gap]

**Intuition:** DE agent expects TFP to fall by a lot, (in excess of reality)

\[ \implies \text{Persistent drop in consumption} \]

NK model calibration: Galí (2015) textbook \((\beta = 0.99, \kappa = 0.05)\) + DE parameter \((\theta = 1)\). See: L'Huillier, Singh and Yoo (2021)
Reversal to Rationality: Output Gap after negative TFP shock

Taylor Rule: $\hat{i}_t = 1.50\pi_t + 0.5\bar{x}_t$; TFP process $a_t = 0.9a_{t-1} + \epsilon_t$

Intuition: DE agent expects TFP to fall by a lot, (in excess of reality) $\implies$ Persistent drop in consumption

NK model calibration: Gali (2015) textbook ($\beta = 0.99$, $\kappa = 0.05$) + DE parameter ($\theta = 1$). See: L'Huillier, Singh and Yoo (2021), Equal weights on $J = 1$ and $J = 2$ reference periods
Empirical IRFs to Romer and Romer (2004) shocks: monthly frequency

\[ x_{t+h} = c_h + \tau_h t + \beta_h^M \varepsilon_t^{RR} + \Gamma Z_t + \eta_{t+h}; \quad h = 0, ..., H \]

- Jordá (2005) Local Projections
- \( \beta_h^M \) directly plots the causal effect of monetary shock on \( x \) at horizon \( h \)
- Monthly data from FRED (INDPRO, CPIAUCSL, FEDFUNDS)
- \( Z_t \) includes 12 (48) monthly lags of the LHS (\( x_{t-i} \)) and the RR shocks variable
- \( t \): is a linear time-trend
- Shaded areas denote 95% robust HAC standard error bands
Empirical IRFs to Romer and Romer (2004) shocks: monthly frequency

- **Industrial Production**
  - Months: 0 to 120
  - Percent: -4 to 4

- **CPI**
  - Months: 0 to 120
  - Percent: -2 to 2

- **Federal Funds Rate**
  - Months: 0 to 120
  - Percent: -4 to 4
Empirical IRFs to Romer and Romer (2004) shocks: quarterly frequency

\[ x_{t+h} = c_h + \tau_h t + \beta_h^Q \varepsilon_t^{RR} + \Gamma Z_t + \eta_{t+h}; \quad h = 0, \ldots, H \]

- Jordá (2005) Local Projections
  - \( \beta_h^Q \) directly plots the causal effect of monetary shock on \( x \) at horizon \( h \)
- Quarterly data from FRED (GDPC1, CPIAUCSL, FEDFUNDS, GDPIC1, B230RCQ173SBEA, PCDG/DDURRD3Q086SBEA) + McKay and Wieland (2021) replication package (qres, qall, qnonall)
  - \( Z_t \) includes 16 quarterly lags of the LHS \( (x_{t-i}) \) and RR shocks
  - \( t \): is a linear time-trend
  - Shaded areas denote 95% robust HAC standard error bands
Empirical IRFs to Romer and Romer (2004) shocks: quarterly frequency
Empirical IRFs to Romer and Romer (2004) shocks: quarterly frequency

- Residential Expenditure
- Consumer Durable Expenditure
- Durable Expenditure
- Non-Durable Expenditure
Beaudry-Portier /McKay-Wieland stories require a non-linear model for boom-bust. This is a linear model!

DSGE model matches the survey expectations IRF. Very nice!

Monthly IRFs noisy: is there a frequency implication in the model?

Boom-bust story seems to be largely residential and non-durable expenditure?

What is the model fit of RE vs DE based only on 20 quarters IRF?

Are IRFs beyond horizon 20, from 120 quarters data, reliable?

Jordá, Singh & Taylor (2021): The Long-run Effects of Monetary Policy,

Bernanke & Mihov (1998): The Liquidity Effect and Long-Run Neutrality

DE ≈ endogenous news shocks (compare with exogenous news/noise shocks)?