For Online Publication

Output Hysteresis and Optimal Monetary Policy

Appendix

Vaishali Garga and Sanjay R. Singh[†]

[†] Garga: vaishali.garga@bos.frb.org, Research Department, Federal Reserve Bank of Boston, MA 02210.
Singh: sjrsingh@ucdavis.edu. Department of Economics, University of California, Davis CA 95616.

Appendix A. Competitive Equilibrium

Definition Appendix A.1 (Competitive Equilibrium)**.** The competitive equilibrium is defined as a sequence of 9 quantities $\{C_t,\, z_t,\, V_t,\, \Gamma_t,\, Y_t^G,\, Y_t,\, RD_t,\, L_t,\, A_t\}$ and 7 prices $\{i_t,\, Q_{t,t+1},\, P_t,\, W_t,\, K_t,\, F_t,\, \pi_{W,t},\}$ which satisfy the following 16 equations, for a given sequence of exogenous shocks $\{\varepsilon_t,\, \xi_t,\, M_t,\, \lambda_{w,t}\}$ and exogenously specified policy variables $\{\tau_t^b,\, \tau_t^r,\, \tau_t^p,\, \tau_t^w\}$.

1. Euler Equation and Stochastic Discount Factor

$$1 = \beta \mathbb{E}_t \left[\frac{C_{t+1}^{-1}}{C_t^{-1}} (1+i_t) \frac{P_t}{P_{t+1}} (1-\tau_t^b) \right] + \xi_t C_t$$

$$Q_{t,t+1} = \beta \frac{C_{t+1}^{-1}}{C_t^{-1}} \frac{P_t}{P_{t+1}}$$

2. Endogenous Growth Block

$$(1-\tau_t^r) P_t \delta \varrho z_t^{\varrho-1} = \mathbb{E}_t Q_{t,t+1} V_{t+1}(A_{t+1})$$

$$V_t(A_t) = \Gamma_t + (1 - z_t - \eta) \mathbb{E}_t Q_{t,t+1} V_{t+1}(A_t)$$

$$\Gamma_t = ((1-\tau^p)\zeta - 1) \left(\frac{\alpha}{\zeta} \right)^{\frac{1}{1-\alpha}} P_t M_t L_t A_t$$

where $\zeta \equiv \min\left(\gamma^{1-\alpha}, \frac{1}{(1-\tau_t^p)\alpha} \right)$, and $\gamma > 1$.

3. Wage Setting frictions

$$\frac{K_t}{F_t} = \left(\frac{1 - \theta_w (\pi_t^w)^{\frac{1}{\lambda_{w,t}}}}{1-\theta_w} \right)^{-\lambda_{w,t} + (1+\lambda_{w,t})\nu}$$

$$K_t = \omega (1+\lambda_{w,t}) L_t^{1+\nu} + \theta_w \beta \bar{\Pi}_W^{-\frac{(1+\lambda_{w,t+1})(1+\nu)}{\lambda_{w,t+1}}} \Pi_{W,t+1}^{\frac{(1+\lambda_{w,t+1})(1+\nu)}{\lambda_{w,t+1}}} K_{t+1}$$

$$F_t = (1+\tau_t^w) L_t C_t^{-1} \frac{W_t}{P_t} + \theta_w \beta \bar{\Pi}_W^{\frac{-1}{\lambda_{w,t+1}}} \Pi_{W,t+1}^{\frac{1}{\lambda_{w,t+1}}} F_{t+1}$$

$$\Pi_{W,t} = \frac{W_t}{W_{t-1}}$$

4. Law of motion of productivity

$$A_t = A_{t-1} + z_{t-1}(\gamma - 1) A_{t-1}$$

5. Market Clearing Conditions and Production Technologies

$$Y_t^G = \left(\frac{\alpha}{\zeta} \right)^{\frac{\alpha}{1-\alpha}} M_t L_t A_t$$

$$RD_t = \delta z_t^{\varrho} A_t$$

$$(1 - \frac{\alpha}{\zeta})Y_t^G = Y_t$$

$$Y_t = C_t + RD_t$$

$$W_t = (1-\alpha)\left(\frac{\alpha}{\zeta}\right)^{\frac{\alpha}{1-\alpha}} M_t A_t P_t$$

6. Monetary Policy Rule

$$1 + i_t = \max\left(1, (1+i_{ss})\left(\frac{\pi_{W,t}}{\bar{\pi}_W}\right)^{\phi_\pi}\left(\frac{L_t}{\bar{L}}\right)^{\phi_y}\varepsilon_t^i\right); \quad \phi_\pi > 1; \phi_y > 0$$

Stationarizing the System

The competitive equilibrium defined above is non-stationary. Specifically, consumption, output, nominal wage, are co-integrated with TFP level A_t. We normalize the variables as follows:

$$c_t \equiv \frac{C_t}{A_t}; y_t \equiv \frac{Y_t}{A_t}; y_t^G \equiv \frac{Y_t^G}{A_t}; rd_t \equiv \frac{RD_t}{A_t}; \tilde{\Gamma}_t \equiv \frac{\Gamma_t}{P_t A_t}; w_t \equiv \frac{W_t}{P_t A_t}$$

Further note that because of the linearity assumption in the production of final goods, the Value function is a linear function in productivity with which an entrepreneur enters the sector:

$$\tilde{V}_t = \frac{V_t}{A_t} = \tilde{\Gamma}_t + (1 - z_t - \eta)\mathbb{E}_t Q_{t,t+1}\tilde{V}_{t+1}$$

where $\tilde{V}$ is normalized by the productivity with which the entrepreneur enters the sector. Finally the growth rate of productivity, determined in period t, is given by

$$(1 + g_{t+1}) = 1 + z_t(\gamma - 1)$$

Remaining variables are stationary.

Definition Appendix A.2 (Normalized Competitive Equilibrium)**.** The *normalized competitive equilibrium* is defined as a sequence of 9 stationary quantities $\{c_t, \tilde{V}_t, \tilde{\Gamma}_t, y_t^G, y_t, rd_t, L_t, g_{t+1}, z_t\}$ and 6 stationary prices $\{i_t, w_t, K_t, F_t, \pi_{W,t}, \Pi_t\}$ which satisfy the following 15 equations, for a given sequence of exogenous shocks $\{\varepsilon_t, \xi_t, M_t, \lambda_{w,t}\}$ and exogenously specified policy variables $\{\tau_t^b, \tau_t^r, \tau_t^p, \tau_t^w\}$.

1. Euler Equation and Stochastic Discount Factor

$$1 = \beta\mathbb{E}_t\left[\frac{c_{t+1}^{-1}(1+g_{t+1})^{-1}}{c_t^{-1}}\frac{1+i_t}{\pi_{t+1}}(1-\tau_t^b)\right] + \xi_t' c_t$$

where $\xi_t' = \xi_t A_t$

2. Endogenous Growth Block

$$(1-\tau_t^r)\delta\varrho z_t^{\varrho-1} = \mathbb{E}_t \frac{c_{t+1}^{-1}(1+g_{t+1})^{-1}}{c_t^{-1}}\gamma\tilde{V}_{t+1}$$

$$\tilde{V}_t = \tilde{\Gamma}_t + (1-z_t-\eta)\beta\mathbb{E}_t \frac{c_{t+1}^{-1}(1+g_{t+1})^{-1}}{c_t^{-1}}\tilde{V}_{t+1}$$

$$\tilde{\Gamma}_t = ((1-\tau^p)\zeta - 1)\left(\frac{\alpha}{\zeta}\right)^{\frac{1}{1-\alpha}} M_t L_t$$

where $\zeta \equiv \min\left(\gamma^{1-\alpha}, \frac{1}{(1-\tau_t^p)\alpha}\right)$, and $\gamma > 1$.

3. Wage Setting frictions

$$\frac{K_t}{F_t} = \left(\frac{1-\theta_w(\pi_t^w)^{\frac{1}{\lambda_{w,t}}}}{1-\theta_w}\right)^{-\lambda_{w,t}+(1+\lambda_{w,t})\nu}$$

$$K_t = \omega(1+\lambda_{w,t}){L_t}^{1+\nu} + \theta_w\beta\bar{\Pi}_W^{-\frac{(1+\lambda_{w,t+1})(1+\nu)}{\lambda_{w,t+1}}}\Pi_{W,t+1}^{\frac{(1+\lambda_{w,t+1})(1+\nu)}{\lambda_{w,t+1}}}K_{t+1}$$

$$F_t = (1+\tau_t^w)L_t c_t^{-1} w_t + \theta_w\beta\bar{\Pi}_W^{\frac{-1}{\lambda_{w,t+1}}}\Pi_{W,t+1}^{\frac{1}{\lambda_{w,t+1}}}F_{t+1}$$

$$\pi_{w,t} = \frac{w_t}{w_{t-1}}(1+g_t)\pi_t$$

4. Productivity growth rate

$$(1+g_{t+1}) = 1 + z_t(\gamma-1)$$

5. Market Clearing Conditions and Production Technologies

$$y_t^G = \left(\frac{\alpha}{\zeta}\right)^{\frac{\alpha}{1-\alpha}} M_t L_t$$

$$rd_t = \delta z_t^{\varrho}$$

$$(1-\frac{\alpha}{\zeta})y_t^G = y_t$$

$$y_t = c_t + rd_t$$

$$w_t = (1-\alpha)\left(\frac{\alpha}{\zeta}\right)^{\frac{\alpha}{1-\alpha}} M_t$$

6. Monetary Policy Rule

$$1+i_t = \max\left(1, (1+i_{ss})\left(\frac{\pi_{W,t}}{\bar{\pi}_W}\right)^{\phi_\pi}\left(\frac{L_t}{\bar{L}}\right)^{\phi_y}\varepsilon_t^i\right); \quad \phi_\pi > 1; \phi_y > 0$$

Steady State

Six variables z, g, V, L, C, Y solve the following six equations

1. Endogenous Growth Equation

$$(1-\tau^r)\varrho z^{\varrho-1} = \frac{\beta}{1+g}\frac{\gamma\tilde{V}}{\delta}$$

2. Value Function

$$\tilde{V} = \frac{((1-\tau^p)\zeta - 1)\left(\frac{\alpha}{\zeta}\right)^{\frac{1}{1-\alpha}} L}{1+g-\beta(1-z-\eta)}(1+g)$$

3. Intra-temporal Labor Supply condition

$$\omega L^{\nu} c = (1-\alpha)\left(\frac{\alpha}{\zeta}\right)^{\frac{\alpha}{1-\alpha}}$$

4. Aggregate Production Function

$$y = \left(1-\frac{\alpha}{\zeta}\right)\left(\frac{\alpha}{\zeta}\right)^{\frac{\alpha}{1-\alpha}} L$$

5. Resource Constraint

$$c + \delta z^{\varrho} = y$$

6. Growth equation (law of motion of productivity)

$$g = z(\gamma - 1)$$

Other steady state variables can be backed out after solving this system. We look for steady state such that $z \in (0, 1-\eta)$ and $c \geq 0$. In what follows, we will set $\eta = 0$ to derive formulas. It is relatively straightforward to extend the system to $\eta > 0$.

Definition Appendix A.3 (Approximate Equilibrium)**.** An approximate competitive equilibrium in this economy with endogenous growth is defined as a sequence of variables $\{\hat{\pi}_t^w,\, \hat{c}_t,\, \hat{y}_t,\, \hat{g}_{t+1},\, \hat{i}_t,\, \hat{L}_t,\, \hat{w}_t,\, \hat{\pi}_t,\, \hat{V}_t\}$ which satisfy the following equations, for a given sequence of exogenous shocks $\{\hat{\xi}_t,\, \hat{M}_t,\, \hat{\epsilon}_t^i,\, \hat{\lambda}_{wt}\}$.

Aggregate Demand:

$$-\left(\mathbb{E}_t\hat{c}_{t+1} - \hat{c}_t + \hat{g}_{t+1}\right) + \hat{i}_t - \mathbb{E}_t\hat{\pi}_{t+1} + \hat{\xi}_t = 0 \tag{A.1}$$

Endogenous growth equations:

$$(\varrho-1)\eta_g\hat{g}_{t+1} = -(\mathbb{E}_t\hat{c}_{t+1} - \hat{c}_t + \hat{g}_{t+1}) + \mathbb{E}_t\hat{V}_{t+1} \tag{A.2}$$

$$\hat{V}_t = \eta_y\hat{y}_t - \eta_z\hat{g}_{t+1} - \eta_q(\mathbb{E}_t\hat{c}_{t+1} - \hat{c}_t + \hat{g}_{t+1}) + \eta_q\mathbb{E}_t\hat{V}_{t+1} \tag{A.3}$$

where $\eta_y = 1 - \frac{(1-z)\beta}{1+g} > 0$, $\eta_z = \frac{\beta}{\gamma-1} > 0$, $\eta_q = \frac{(1-z)\beta}{1+g} > 0$

Market clearing:

$$\frac{c}{y}\hat{c}_t + \frac{\mathbb{R}}{y}\varrho\eta_g\hat{g}_{t+1} = \hat{y}_t \tag{A.4}$$

$$\hat{y}_t = \hat{M}_t + \hat{L}_t \tag{A.5}$$

Wage setting:

$$\hat{\pi}_t^w = \tilde{\beta}\mathbb{E}_t\hat{\pi}_{t+1}^w + \kappa_w[\hat{c}_t + \nu\hat{L}_t - \hat{w}_t] + \kappa_w\hat{\lambda}_{wt} \tag{A.6}$$

$$\hat{w}_t = \hat{M}_t \tag{A.7}$$

$$\hat{\pi}_t^w = \hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_t + \hat{g}_t \tag{A.8}$$

where $\kappa_w \equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\nu\left(1+\frac{1}{\lambda_w}\right))} > 0$

Monetary policy rule:

$$\hat{i}_t = \max\left(-\frac{\bar{i}}{1+\bar{i}}, \phi_\pi\hat{\pi}_t^w + \phi_y\hat{L}_t + \hat{\varepsilon}_t^i\right) \tag{A.9}$$

Appendix B. Local Determinacy with one-period patent and exogenous nominal wages

To analytically characterize the determinacy condition, we make following two assumptions, that we will refer to as T1 and T2:

Assumption T1 Deterministic patent length of one period: Upon successful innovation, the entrepreneur gets the monopoly right over production of intermediate good in the following period $t+1$. if a randomly selected entrepreneur fails to innovate at $t+1$, the planner selects a producer from a fringe of measure zero to produce using period $t+1$'s productivity in the following period. The first order condition reported in the endogenous growth block above is modified to:

$$(1-\tau_t^r)P_t\delta\varrho z_t^{\varrho-1} = \mathbb{E}_t Q_{t,t+1}\Gamma_{t+1}(A_{t+1})$$

where $\Gamma_t(A_t) = ((1-\tau^p)\zeta - 1)\left(\frac{\alpha}{\zeta}\right)^{\frac{1}{1-\alpha}} P_t M_t L_t A_t$, $\zeta \equiv \min\left(\gamma^{1-\alpha}\frac{1}{(1-\tau_t^p)\alpha}\right)$, and $\gamma > 1$.

Assumption T2 Perfect nominal wage rigidity with indexation: Nominal wages are assumed to evolve :

$$W_t = \bar{\pi}_W W_{t-1}$$

This equation replaces the wage Phillips curve derived above.

Both these assumptions allow us to analytically derive determinacy conditions, at and away from the ZLB.

Appendix B.1. Away from ZLB

Under T1 and T2, the approximate equilibrium (assuming no shocks) is given by:

$$\hat{L}_t = \mathbb{A}_1 \mathbb{E}_t \hat{L}_{t+1} + \mathbb{A}_2 \mathbb{E}_t \hat{g}_{t+2}$$

$$\hat{g}_{t+1} = \mathbb{A}_3 \mathbb{E}_t \hat{L}_{t+1} + \mathbb{A}_4 \mathbb{E}_t \hat{g}_{t+2}$$

where $\mathbb{A}_1 \equiv \frac{\frac{y}{c} + \frac{\left(\frac{y}{c}-1\right)\varrho\eta_g}{(\varrho-1)\eta_g+1}}{\frac{y}{c}+\phi\left(1+\frac{\left(\frac{y}{c}-1\right)}{(\varrho-1)\eta_g+1}\right)}$, $\mathbb{A}_2 \equiv \frac{\left(\frac{y}{c}-1\right)\varrho\eta_g}{\frac{y}{c}+\phi\left(1+\frac{\left(\frac{y}{c}-1\right)}{(\varrho-1)\eta_g+1}\right)}$, $\mathbb{A}_3 \equiv \frac{1-\phi\mathbb{A}_1}{(\varrho-1)\eta_g+1}$, $\mathbb{A}_4 \equiv \frac{-\phi\mathbb{A}_2}{(\varrho-1)\eta_g+1}$, $\eta_g = \frac{1+g}{g}$ and lowercase letter y, c, and g denote steady state values. The system is locally determinate iff following two conditions are met:

$$|\mathbb{A}_1\mathbb{A}_4 - \mathbb{A}_2\mathbb{A}_3| < 1; \quad |\mathbb{A}_1 + \mathbb{A}_4| < 1 + \mathbb{A}_1\mathbb{A}_4 - \mathbb{A}_2\mathbb{A}_3$$

The second condition is met as long as $\phi > 0$. The first condition is met if:

$$\eta_g > \frac{\beta\gamma}{\gamma - 1} > 1$$

This condition also implies that consumption is positive and there is positive R&D investment. Assuming $\varrho = 1$, this condition can be rewritten solely in terms of parameters as in Benigno and Fornaro (2017):

$$1 + \frac{\Psi(\gamma-1)}{\delta} > \frac{\beta\gamma\varpi}{\delta} > 1; \quad \text{where } \Psi \equiv \left(1 - \frac{\alpha}{\zeta}\right)\left(\frac{\alpha}{\zeta}\right)^{\frac{\alpha}{1-\alpha}}; \quad \varpi \equiv (\zeta - 1)\left(\frac{\alpha}{\zeta}\right)^{\frac{1}{1-\alpha}}$$

For a general ϕ, conditional on a steady state with positive consumption and positive R&D investment, following two conditions guarantee determinacy:

$$\phi > 0; \quad \text{and} \quad \eta_g > \frac{\beta\gamma}{\gamma - 1} > 1$$

Appendix B.2. ZLB with two-state Markov Chain

Assuming the two-state Markov Chain where with probability $\mu \in (0, 1)$ the economy continues to stay at the ZLB and with $1 - \mu$ it escapes the ZLB, the system (with T1 and T2) in the short-run ZLB state (S) can be expressed as:

$$\hat{L}^S_t = \mathbb{A}^S_1 \mathbb{E}_t \hat{L}^S_{t+1} + \mathbb{A}^S_2 \mathbb{E}_t \hat{g}^S_{t+2} + \mathbb{Q}^S_1 \xi_S$$

$$\hat{g}^S_{t+1} = \mathbb{A}^S_3 \mathbb{E}_t \hat{L}^S_{t+1} + \mathbb{A}^S_4 \mathbb{E}_t \hat{g}^S_{t+2} + \mathbb{Q}^S_2 \xi_S$$

where $\mathbb{A}_1^S \equiv \mu\left(1+\frac{(1-\frac{c}{y})\varrho\eta_g}{(\varrho-1)\eta_g+1}\right)$, $\mathbb{A}_2^S \equiv -\mu\left(1-\frac{c}{y}\right)\varrho\eta_g$, $\mathbb{A}_3^S \equiv \frac{\mu}{(\varrho-1)\eta_g+1}$, $\mathbb{A}_4^S \equiv 0$, $\eta_g = \frac{1+g}{g}$ and lowercase letter y, c, and g denote steady state values. The system is locally determinate iff:

$$\mu < \frac{((\varrho-1)\eta_g+1)(\gamma-1)}{\beta\gamma}$$

Most calibrations have $\gamma \in (1.05, 1.55)$ and $\beta \in (0.96, 1)$. For values of $\varrho \geq 1.105$, 2% annual steady state growth rate and the above parameter bounds on γ and β, this condition is always satisfied. Given these empirically plausible parameter restrictions, we obtain local determinacy at the ZLB.

Appendix C. Impulse Responses under Taylor rule eq 7

We show the detailed derivation for impulse response under the Taylor rule eq (7) and liquidity demand shock. For monetary policy shock, productivity shock and markup shock, the proof is similar. Assume that the liquidity demand shock follows the AR(1) process:

$$\hat{\xi}_t = \rho_i\hat{\xi}_{t-1} + \hat{\epsilon}_t$$

Guess the solution takes the form:

$$\hat{c}_t = \psi_c\hat{\epsilon}_t; \hat{y}_t = \psi_y\hat{\epsilon}_t; \hat{g}_{t+1} = \psi_g\hat{\epsilon}_t; \hat{\pi}_t^w = \psi_p\hat{\epsilon}_t; \hat{V}_t = \psi_v\hat{\epsilon}_t$$

From Euler equation, we get:

$$(1-\rho_i)\psi_c = -(\phi_\pi - \rho_i)\psi_p - \phi_y\psi_y - 1 \tag{C.1}$$

From the Endogeneous Growth equation:

$$(1-\rho_i)\psi_c + \rho_i\psi_v = [(\varrho-1)\eta_g + 1]\psi_g \tag{C.2}$$

From the Resource constraint:

$$\frac{c}{y}\psi_c + \frac{\mathbb{R}}{y}\varrho\eta_g\psi_g = \psi_y \tag{C.3}$$

From the Wage Phillips curve

$$(1-\beta\rho_i)\psi_p = \kappa_w(\psi_c + \nu\psi_y) \tag{C.4}$$

From the Value function:

$$(1-\eta_V\rho_i)\psi_v = \eta_y\psi_y - (\eta_z+\eta_q)\psi_g + \eta_q(1-\rho_i)\psi_c \tag{C.5}$$

From equations C.2, C.3, and C.5, we can find a relation between ψ_c and ψ_y. Rest of the system is pretty standard NK system where we can solve for ψ_p and ψ_y from equations C.1 and C.4 using:

$$\psi_c = \frac{\frac{1-\eta_V\rho_i}{\rho_i}\frac{(\varrho-1)\eta_g+1}{\frac{\mathbb{R}}{y}\varrho\eta_g} + \frac{\eta_z+\eta_q}{\frac{\mathbb{R}}{y}\varrho\eta_g} - \eta_Y}{\left[\frac{1-\eta_V\rho_i}{\rho_i}\frac{(\varrho-1)\eta_g+1}{\frac{\mathbb{R}}{y}\varrho\eta_g}\frac{c}{y} + (1-\rho_i)\right] + \frac{\eta_z+\eta_q}{\frac{\mathbb{R}}{y}\varrho\eta_g}\frac{c}{y} + \eta_q(1-\rho_i)}\psi_y = A_1\psi_y; \quad 0 < A_1 < 1$$

We get:

$$\psi_p = \frac{\kappa(A_1+\nu)}{1-\beta\rho_i}\psi_y \equiv A_2\psi_y$$

And thus:

$$\psi_y = \frac{-1}{(1-\rho_i)A_1 + (\phi_\pi - \rho_i)A_2 + \phi_y} < 0$$

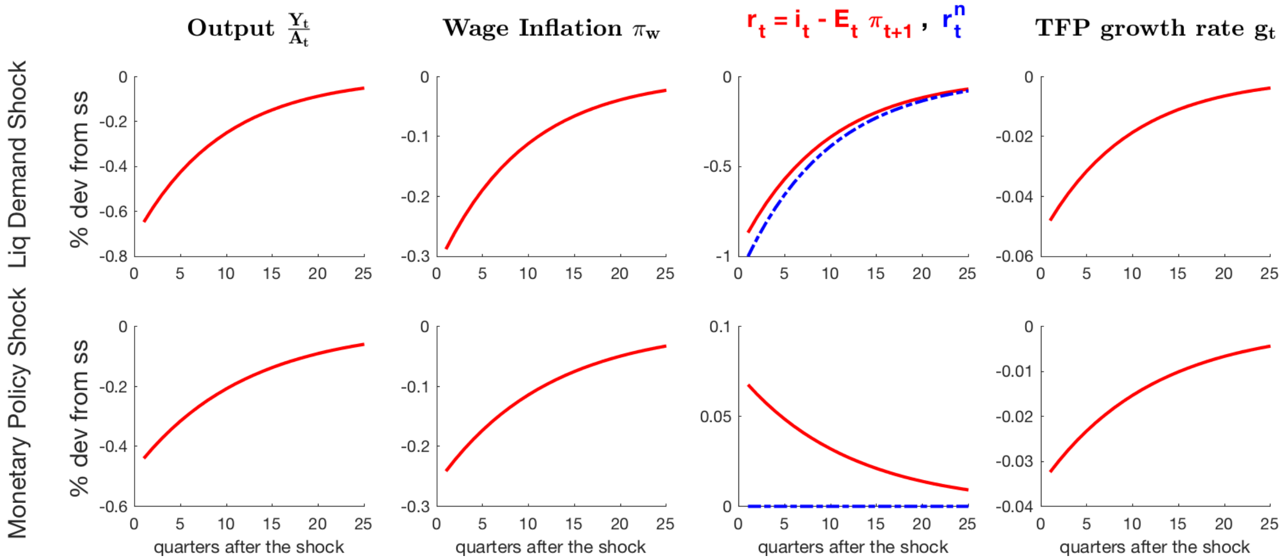
Further, from the resource constraint we find :

$$\psi_g = \frac{\psi_y - \frac{c}{y}\psi_c}{\frac{\mathbb{R}}{y}\varrho\eta_g} = \frac{1-\frac{c}{y}A_1}{\frac{\mathbb{R}}{y}\varrho\eta_g}\psi_y$$

Since $A_1 < 1$, it follows that $A_1 < \frac{y}{c}$. Hence there is a positive co-movement of output and growth rate under liquidity demand shock. Further it must be that the following holds

$$\psi_v = \frac{[(\varrho-1)\eta_g+1]\psi_g - (1-\rho_i)\psi_c}{\rho_i} = \frac{\eta_y\psi_y - (\eta_z+\eta_q)\psi_g + \eta_q(1-\rho_i)\psi_c}{1-\eta_v\rho_i}$$

Figure C.1: Model-Based Impulse Response Functions



Source: Authors' calculations.
Note: The figures illustrate the impulse response functions (IRFs) from the benchmark model presented in Section 2. The IRFs are plotted in response to liquidity demand shock, and monetary policy shock, with persistence 0.9 and 0.92, respectively. Also, ss = steady state.

Row 1 in Figure C.1 plots the impulse responses for normalized output, wage inflation, real interest rate and productivity growth rate for a positive shock to liquidity demand ξ_t, that corresponds to a fall in the annualized natural interest rate of 1 percentage point, and follows an AR(1) process with persistence of 0.90. Monetary policy is assumed to follow a standard Taylor rule (equation 7) with $\phi_\pi = 1.5$ and $\phi_y = 0.5$. This shock to ξ_t increases the household's desire for saving in the risk-free bond, thereby diverting resources away from consumption. Lower anticipated aggregate demand reduces investment in R&D by entrepreneurs, also exerting a drag on productivity growth. Similar dynamics for an AR(1) contractionary monetary policy shock with persistence 0.92, presented in row 2 of Figure C.1. The equilibrium increase in the real interest rate, combined with expectations of lower future aggregate demand, leads to a reduction in R&D investment and, therefore, in TFP growth.

Appendix D. Solution to Social Planner's Problem

Appendix D.1. Social Planner problem I

The Social Planner chooses $\{C_t, L_t, A_{t+1}, z_t\}$ to maximize the welfare function:

$$\max \quad \log C_t - \frac{\omega}{1+\nu} L_t^{1+\nu}$$

subject to the constraints:

$$C_t + RD_t = \alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)A_tL_t$$

$$\frac{A_{t+1}-A_t}{A_t} = (\gamma-1)z_t$$

$$R_t = \delta z_t^{\varrho} A_t$$

$$z_t \geq 0$$

Combining the constraints and using the functional form for R&D Investment, we get:

$$C_t + \delta\left(\frac{A_{t+1}-A_t}{A_t}\frac{1}{\gamma-1}\right)^{\varrho} A_t = \alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)A_tL_t$$

Let λ_t be the Lagrange multiplier on the constraint. Solution to this problem is thus :

$$\lambda_t = \frac{1}{C_t}$$

$$L_t^{\nu}C_t = \alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)$$

$$\frac{-\lambda_t\delta\varrho}{(\gamma-1)^{\varrho}}\left(\frac{A_{t+1}-A_t}{A_t}\right)^{\varrho-1} + \frac{\lambda_{t+1}\beta\delta\varrho}{(\gamma-1)^{\varrho}}\left(\frac{A_{t+2}-A_{t+1}}{A_{t+1}}\right)^{\varrho-1} - \frac{\lambda_{t+1}\beta\delta(\varrho-1)}{(\gamma-1)^{\varrho}}\left(\frac{A_{t+2}-A_{t+1}}{A_{t+1}}\right)^{\varrho} + \alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)L_{t+1}\lambda_{t+1}\beta = 0$$

Since growth rate is defined as $g_{t+1} = \frac{A_{t+1}-A_t}{A_t}$, we can rewrite the above condition as:

$$\frac{C_{t+1}}{C_t}\frac{\delta\varrho}{(\gamma-1)^{\varrho}}g_{t+1}^{\varrho-1} = \frac{\beta\delta\varrho}{(\gamma-1)^{\varrho}}g_{t+2}^{\varrho-1} - \frac{\beta\delta(\varrho-1)}{(\gamma-1)^{\varrho}}g_{t+2}^{\varrho} + \alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)L_{t+1}\beta$$

This can be rewritten as:

$$\frac{C_{t+1}}{C_t} = \beta\left[\left(\frac{g_{t+2}}{g_{t+1}}\right)^{\varrho-1} - \frac{1-\varrho}{\varrho}g_{t+2}\left(\frac{g_{t+2}}{g_{t+1}}\right)^{\varrho-1} + \frac{\alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)}{\delta\varrho}\frac{L_{t+1}(\gamma-1)^{\varrho}}{g_{t+1}^{\varrho-1}}\right]$$

This the Euler equation for R&D investment in the Social Planner's allocation. The right hand side gives the return on R&D investment. Writing the LHS in normalized terms i.e. $C_t = c_tA_t$, we get

$$\frac{c_{t+1}(1+g_{t+1})}{c_t} = \beta\left[\left(\frac{g_{t+2}}{g_{t+1}}\right)^{\varrho-1} - \frac{1-\varrho}{\varrho}g_{t+2}\left(\frac{g_{t+2}}{g_{t+1}}\right)^{\varrho-1} + \frac{\alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)}{\delta\varrho}\frac{L_{t+1}(\gamma-1)^{\varrho}}{g_{t+1}^{\varrho-1}}\right] \quad \text{(D.1)}$$

The (interior) equilibrium (with positive growth) is thus given by the sequence of three variables $\{c_t, L_t, g_{t+1}\}$ such that equation D.1 and following two conditions (intra-temporal labor supply and budget constraint) are satisfied:

$$\omega L_t^{\nu}c_t = \alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha) \quad \text{(D.2)}$$

$$c_t + \delta\left(\frac{g_{t+1}}{\gamma-1}\right)^{\varrho} = \alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)L_t \quad \text{(D.3)}$$

Appendix D.2. Policy Relevant Welfare Function

The representative agent's lifetime welfare function at time t can be rewritten as

$$V_t = \sum_{s=t}^{\infty} \beta^{s-t} \left[\log C_s - v(L_s)\right] = \sum_{s=t}^{\infty} \beta^{s-t} \left[\log c_s - v(L_s) + \frac{\beta}{1-\beta} \log(1+g_{s+1})\right] + \frac{1}{1-\beta} \log A_t$$

We redefine the terms in the square brackets as the policy relevant per period welfare function:

$$\mathbf{W}_t = \log c_t - v(L_t) + \frac{\beta}{1-\beta} \log(1+g_{t+1})$$

Thus the policy relevant lifetime welfare function is given by

$$\mathbb{W}_t = \sum_{s=t}^{\infty} \beta^{s-t} \left[\log c_s - v(L_s) + \frac{\beta}{1-\beta} \log(1+g_{s+1})\right]$$

Appendix D.3. Social Planner problem II

The Social Planner chooses $\{c_t, L_t, g_{t+1}, z_t\}$ to maximize lifetime-policy relevant welfare function:

$$\max \quad \sum_{s=t}^{\infty} \beta^{s-t} \left[\log c_s - \frac{\omega}{1+\nu} L_s^{1+\nu} + \frac{\beta}{1-\beta} \log(1+g_{s+1})\right]$$

subject to

$$c_t + rd_t = \alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)L_t = y_t$$

$$g_{t+1} = z_t(\gamma - 1)$$

$$rd_t = \delta z_t^{\varrho}$$

$$z_t \geq 0$$

Solution (for $z > 0$) is given by:

$$\frac{rd'(z_t)}{c_t} = (\gamma - 1)\frac{\beta}{1-\beta}\frac{1}{1+g_{t+1}}$$

$$\omega L_t^{\nu} c_t = \alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)$$

$$c_t + rd_t = \alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)L_t = y_t$$

$$rd_t = \delta z_t^{\varrho}$$

$$g_{t+1} = z_t(\gamma - 1)$$

Substituting out for research intensity z_t in terms of growth rate and using the functional form for R&D Investment, Solution is given by Intra-temporal labor supply condition eq D.2, Budget constraint eq D.3 and the following R&D investment condition:

$$\varrho\delta\left(\frac{g_{t+1}}{\gamma-1}\right)^{\varrho-1}=(\gamma-1)\frac{\beta}{1-\beta}\frac{c_t}{1+g_{t+1}} \tag{D.4}$$

Appendix D.4. Equivalence of two solutions

It is clear that Euler condition derived in eq D.1 is not as amenable to analytical manipulations as is the corresponding R&D investment condition eq D.4 derived under the modified Social Planner problem II. Remains to be shown that the resulting equilibrium is identical in both scenarios.

In Steady State eq D.1 simplfies to:

$$(1+g)=\beta\left[1-\frac{1-\varrho}{\varrho}g+\frac{\alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)}{\delta\varrho}\frac{L(\gamma-1)^{\varrho}}{g^{\varrho-1}}\right]$$

It is straightforward to show that eq D.4 combined with eq D.3 also yields the above condition. Thus, the solutions are identical at the steady state.[†] As regards the dynamics away from the steady state, eq D.1 can be rewritten as:

$$\frac{c_{t+1}(1+g_{t+1})}{c_t}=\beta\left[\left(\frac{g_{t+2}}{g_{t+1}}\right)^{\varrho-1}(1+g_{t+2})+\frac{c_{t+1}}{\delta\varrho}\frac{(\gamma-1)^{\varrho}}{g_{t+1}^{\varrho-1}}\right] \tag{D.5}$$

From eq D.4, we can write out the RHS of the above equation D.5 as

$$\frac{c_{t+1}(1+g_{t+1})}{c_t}=\left(\frac{g_{t+2}}{g_{t+1}}\right)^{\varrho-1}(1+g_{t+2}) \tag{D.6}$$

Thus, it remains to show that the LHS of two equations D.5 and D.6 are equal. We prove by reduction. Substitute LHS of equation D.6 into RHS of eq D.5 to get:

$$\left(\frac{g_{t+2}}{g_{t+1}}\right)^{\varrho-1}(1+g_{t+2})=\beta\left[\left(\frac{g_{t+2}}{g_{t+1}}\right)^{\varrho-1}(1+g_{t+2})+\frac{c_{t+1}}{\delta\varrho}\frac{(\gamma-1)^{\varrho}}{g_{t+1}^{\varrho-1}}\right]$$

Simple algebraic manipulation yields:

$$(1-\beta)\left(\frac{g_{t+2}}{g_{t+1}}\right)^{\varrho-1}(1+g_{t+2})=\beta\frac{c_{t+1}}{\delta\varrho}\frac{(\gamma-1)^{\varrho}}{g_{t+1}^{\varrho-1}}$$

[†]

$$\frac{\beta c}{\varrho\delta}\frac{(\gamma-1)^{\varrho}}{g^{\varrho-1}}=(1-\beta)(1+g);\quad c=\alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)L-\delta\left(\frac{g}{\varrho-1}\right)^{\varrho}$$

. These yield the above Euler equation.

which can be simplified to yield:

$$\frac{1-\beta}{\beta}\frac{\varrho\delta}{\gamma-1}\left(\frac{g_{t+2}}{\gamma-1}\right)^{\varrho-1}(1+g_{t+2})=c_{t+1}$$

which is true since it is eq D.4 forwarded by one period. Since we do not use the labor-supply intra-temporal condition to show the equivalence between the two solutions under flexible wages, the two approaches are also equivalent under nominal wage rigidities which introduces a wedge in the labor-supply intra-temporal condition.

Appendix D.5. Efficient Steady State

Efficient Steady State is given by following system of equations in three variables c, L, g:

$$L=\left[\frac{\alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)}{\omega c}\right]^{\frac{1}{\nu}};\quad c=\frac{\varrho\delta}{\gamma-1}\frac{1-\beta}{\beta}(1+g)\left(\frac{g}{\gamma-1}\right)^{\varrho-1}$$

$$c+\delta\left(\frac{g}{\gamma-1}\right)^{\varrho}=\alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)L$$

When $\varrho=1$, the solution is given by a fixed point of the following equation:

$$\chi_1(1+g)+\frac{\delta}{\gamma-1}g=\left[\frac{\alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)}{\omega\chi_1(1+g)}\right]^{\frac{1}{\nu}};\quad \text{where } \chi_1\equiv\frac{\delta}{\gamma-1}\frac{1-\beta}{\beta}$$

The LHS is a linear monotonically increasing function of g. RHS is a monotonically decreasing function of g. By single crossing, one can show that there is a unique locally determinate solution for a given condition on χ_1. For higher values of ϱ, numerically we verify local determinacy.

Appendix D.6. Unconventional Policy away from the ZLB: Implementable Allocation

Now we show that the first-best equilibrium allocation can be implemented as the competitive equilibrium using the time-varying fiscal and monetary instruments - nominal interest rate i_t, Tax on interest income τ_t^b, Tax on intermediate goods τ_t^p, Research subsidy for entrepreneurs τ_t^r and Labor tax for household τ_t^w as follows:

$$\tau_t^p=1-\frac{1}{\alpha}$$

$$\tau_t^w=\lambda_{w,t}$$

$$1-\tau_t^r=\frac{1-\beta}{c_{t+1}}(1+g_{t+1})\frac{\gamma}{\gamma-1}\tilde{V}_{t+1}$$

$$\tau_t^b = 0$$

and the nominal interest rate is set such that $W_t = \bar{\pi}_W^t W_{-1}$ - consistent with perfect nominal wage inflation targeting.

Proof. Follows from comparing the system of equations derived under first-best allocation in Appendix C.3 and the (normalized) competitive equilibrium defined in Definition A.2. □

Appendix D.7. Unconventional Fiscal Policy at the ZLB: Implementable Allocation

At the zero lower bound, the nominal interest rate is stuck at 0. However, the first best can still be implemented using the tax subsidy on interest income to offset the ZLB shock. The fiscal instruments are used are as follows: Tax on interest income τ_t^b, Tax on intermediate goods τ_t^p, Research subsidy for entrepreneurs τ_t^r and Labor tax for household τ_t^w as follows:

$$\tau_t^b = \frac{\xi_t'}{\beta c_{t+1}^{-1}(1+g_{t+1})^{-1}} \frac{\pi_{t+1}}{1+i_t} \tag{D.7}$$

$$\tau_t^p = 1 - \frac{1}{\alpha} \tag{D.8}$$

$$\tau_t^w = \lambda_{w,t} \tag{D.9}$$

$$1 - \tau_t^r = \frac{1-\beta}{c_{t+1}}(1+g_{t+1})\frac{\gamma}{\gamma-1}\tilde{V}_{t+1} \tag{D.10}$$

Proof. Follows from comparing the system of equations derived under first-best allocation in Appendix Appendix D.3 and the (normalized) competitive equilibrium defined in Definition A.2. □

As in Correia et al. (2013), it can be shown that the resulting equilibrium is revenue-neutral and time-consistent.

We can re-define the first-best allocation as the equilibrium allocation defined in Definition 1 such that the government provides the time-varying fiscal and monetary instruments listed in eq D.7-D.10.

Appendix D.8. Approximate First-Best Equilibrium

We log-linearize the non-linear equilibrium conditions around the non-stochastic efficient steady state. Approximate first-best equilibrium is given by a sequence of 4 quantities: $\{\hat{L}_t^*, \hat{c}_t^*, \hat{y}_t^*, \hat{g}_{t+1}^*\}$ that solve the following equations for a given exogenous process of shocks $\hat{M}_t$:

$$\nu \hat{L}_t^* + \hat{c}_t^* = \hat{M}_t \tag{D.11}$$

$$(\varrho-1)\eta_g \hat{g}^*_{t+1} = \hat{c}^*_t - \hat{g}^*_{t+1} \tag{D.12}$$

$$\frac{c}{y}\hat{c}^*_t + \frac{rd}{y}\varrho\eta_g \hat{g}^*_{t+1} = \hat{y}^*_t \tag{D.13}$$

$$\hat{M}_t + \hat{L}^*_t = \hat{y}^*_t \tag{D.14}$$

Efficient solution

The above system can be solved to derive the following closed form solution:

$$\hat{g}^*_{t+1} = \psi^*_g \hat{M}_t; \quad \hat{c}^*_t = \psi^*_c \hat{M}_t; \quad \hat{y}^*_t = \psi^*_y \hat{M}_t; \quad \hat{L}^*_t = \psi^*_l \hat{M}_t$$

where $\psi^*_g = \frac{1+\nu}{(\nu\frac{c}{y}+1)((\varrho-1)\eta_g+1)+\varrho\frac{rd}{y}\varrho\eta_g} > 0$,
$0 < \psi^*_c = ((\varrho-1)\eta_g + 1)\psi^*_g < 1$,
$\psi^*_y = \frac{c}{y}\psi^*_c + \frac{rd}{y}\varrho\eta_g\psi^*_g > 0$, and
$\psi^*_l = \frac{1-\psi^*_c}{\nu} > 0$

Appendix D.9. Time-t vs. time-0 flexibility

There are two concepts of price *flexibility* in the presence of a pre-determined state variable. One is the Neiss and Nelson (2003) definition of flexible wages, under which wages have been set flexibly since time 0 and remain flexible indefinitely. Wages set under this concept are called time-0 flexible wages. Second concept of flexibility is the Woodford (2003, Ch. 5)'s definition where wages are set flexibly in the current and future periods taking as given the current period value of the state variable. Wages set under this concept are called time-t flexible wages. Based on two concepts of flexible wages, there are *time-0 first best*, *time-0 natural rate*, *time-t first best* and *time-t natural rate* allocations. We formally define each of these shortly. In the main text, to avoid clutter of notation, we used *first best* allocation for time-0 first best allocation and *natural rate* for time-0 natural rate allocation. For the ease of exposition, we referred to time-0 flexible wages as *flexible wages.*

Whether potential output is endogenous or not depends on the precise definition. We defined *potential output* as the level of output that coincides with the time-t first-best allocation. We believe this is more appealing definition than the one based on time-0 concept because it coincides with the maximum non-inflationary output an economy can produce at a given time with efficient use of resources.

Since the normalized equilibrium can be written without any reference to the level of productivity A_t, the normalized allocations based on the two flexibility concepts coincide.

Definition Appendix D.1 (normalized natural rate allocation)**.** The *normalized natural rate allocation* is given by a sequence of variables $\{\hat{c}^f_t, \hat{y}^f_t, \hat{g}^f_{t+1}, \hat{V}^f_t\}$ such that these satisfy the following equations for a given sequence of shocks $\{\hat{\xi}_t, \hat{\varepsilon}^i_t, \hat{M}_t, \hat{\lambda}_{w,t}\}$:

$$\hat{c}_t^f + \nu \hat{y}_t^f - (1+\nu)\hat{M}_t) + \hat{\lambda}_{wt} = 0 \tag{D.15}$$

$$\frac{c}{y}\hat{c}_t^f + \frac{rd}{y}\varrho\eta_g \hat{g}_{t+1}^f = \hat{y}_t^f \tag{D.16}$$

$$(\varrho - 1)\eta_g \hat{g}_{t+1}^f = -(\mathbb{E}_t \hat{c}_{t+1}^f - \hat{c}_t^f + \hat{g}_{t+1}^f) + \mathbb{E}_t \hat{y}_{t+1}^f \tag{D.17}$$

$$\hat{V}_t^f = \eta_y \hat{y}_t^f - \eta_z \hat{g}_{t+1}^f - \eta_q(\mathbb{E}_t \hat{c}_{t+1}^f - \hat{c}_t^f + \hat{g}_{t+1}^f) + \eta_V \mathbb{E}_t \hat{V}_{t+1}^f \tag{D.18}$$

In other words, if $x_t^f = [\hat{c}_t^f, \hat{y}_t^f, \hat{g}_{t+1}^f, \hat{L}_t^f, \hat{V}_t^f]$ is vector of endogenous variables and $\epsilon_t = [\hat{\xi}_t, \hat{\varepsilon}_t^i, \hat{M}_t, \hat{\lambda}_{w,t}]$ is a vector of shocks, then there is a unique flexible allocation independent of history of nominal distortions and is given by the solution of following rational expectations system:

$$Fx_{t+1}^f + Gx_t^f + H\epsilon_t = 0 \tag{D.19}$$

where F, G, and H are matrices of coefficients corresponding to definition A.2. Using a standard rational expectations solution method, the system can be solved as:

$$x_t^f = M\epsilon_t \tag{D.20}$$

Therefore the major difference that these two flexibility concepts generate in the context of our framework is that under time-0 flexibility setting, productivity $A^{f,-\infty}$ is a hypothetical construct that would have occurred had prices and wages been flexible since the beginning of time. Under time-t flexibility, the level of productivity $A^{f,t}$ is the pre-determined level of productivity corresponding to the data A^{data}. Following are the law of motions of the two productivity concepts:

$$A_{t+1}^{f,-\infty} = A_t^{f,-\infty}(1 + g_{t+1}^f)$$

$$A_{t+1}^{f,t} = A_t^{data}(1 + g_{t+1}^f)$$

where $A_t^{f,-\infty}$ is the level of productivity under flexible wages at time t when wages have been flexible since the infinite past. A_t^{data} is the level of productivity given by the Definition A.1 of the competitive equilibrium and g_{t+1}^f is the flexible-wage productivity growth rate solved in the system D.20.

We further emphasize that the distinction between two natural rate concepts defined in our framework is different from that imposed in exogenous growth environments with capital investment (Edge 2003). In our benchmark endogenous growth model, the natural rate of interest is always same under the two concepts of flexibility. Only the levels of productivity and output differ. Importantly, this difference in levels may be permanent depending on the central bank's policy rule. In contrast, the introduction of capital investment introduces a temporary difference in the levels of capital, output as well as the interest rates depending on the nature of flexibility assumed. In those models, there is no medium or long-run difference between various concepts as capital always returns to initial steady state value. Hence, in contrast to the setups in Neiss and Nelson (2003) and Woodford (2003), the potential output is an endogenous object even in the long-run in our framework.

Appendix D.9.1. time- 0 allocations

We can therefore define the time-0 allocations as follows:

Definition Appendix D.2 (time-0 first-best allocation)**.** The *time-0 first best allocation* is defined as sequence of variables $\{Y_t^{*,-\infty},\ A_{t+1}^{*,-\infty},\ C_t^{*,-\infty},\ \hat{c}_t^*,\ \hat{y}_t^*,\ \hat{g}_{t+1}^*,\ \hat{L}_t^*\}$ which satisfy the equations D.11- D.14 and the following equations, given a sequence of shocks $\{\hat{\xi}_t, \hat{\varepsilon}_t^i, \hat{M}_t, \hat{\lambda}_{w,t}\}$ and initial level of productivity A_0:

$$A_{t+1}^{*,-\infty} = A_t^{*,-\infty}(\hat{g}_{t+1}^* + \log(1+g_{ss}))$$

$$Y_t^{*,-\infty} = A_t^{*,-\infty}(\hat{y}_t^* + \log y_{ss})$$

$$C_t^{*,-\infty} = A_t^{*,-\infty}(\hat{c}_t^* + \log c_{ss})$$

Definition Appendix D.3 (time-0 natural rate allocation)**.** The *time-0 natural rate allocation* is defined as sequence of variables $\{Y_t^{f,-\infty}, A_{t+1}^{f,-\infty}, C_t^{f,-\infty}, \hat{c}_t^f, \hat{y}_t^f, \hat{g}_{t+1}^f, \hat{V}_t^f\}$ which satisfy the equations D.15- D.18 and the following equations, given a sequence of shocks $\{\hat{\xi}_t, \hat{\varepsilon}_t^i, \hat{M}_t, \hat{\lambda}_{w,t}\}$ and initial level of productivity A_0:

$$A_{t+1}^{f,-\infty} = A_t^{f,-\infty}(\hat{g}_{t+1}^f + \log(1+g_{ss}))$$

$$Y_t^{f,-\infty} = A_t^{f,-\infty}(\hat{y}_t^f + \log y_{ss})$$

$$C_t^{f,-\infty} = A_t^{f,-\infty}(\hat{c}_t^f + \log c_{ss})$$

Appendix D.9.2. time- t allocations

Similarly, we define the time-t allocations as follows:

Definition Appendix D.4 (time-t first-best allocation)**.** The *time-t first best allocation* is defined as sequence of variables $\{Y_t^{*,t},\ A_{t+1}^{*,t},\ C_t^{*,t},\ \hat{c}_t^*,\ \hat{y}_t^*,\ \hat{g}_{t+1}^*,\ \hat{L}_t^*\}$ which satisfy the equations D.11- D.14 and the following equations, given a sequence of shocks $\{\hat{\xi}_t, \hat{\varepsilon}_t^i, \hat{M}_t, \hat{\lambda}_{w,t}\}$ and the actual level of productivity at date t, A_t^{data}:

$$A_{t+1}^{*,t} = A_t^{data}(\hat{g}_{t+1}^* + \log(1+g_{ss}))$$

$$Y_t^{*,t} = A_t^{data}(\hat{y}_t^* + \log y_{ss})$$

$$C_t^{*,t} = A_t^{data}(\hat{c}_t^* + \log c_{ss})$$

Definition Appendix D.5 (time-t natural rate allocation)**.** The *time-t natural rate allocation* is defined as sequence of variables $\{Y_t^{f,t},\ A_{t+1}^{f,t},\ C_t^{f,t},\ \hat{c}_t^f,\ \hat{y}_t^f,\ \hat{g}_{t+1}^f,\ \hat{V}_t^f\}$ which satisfy the equations D.15- D.18 and the following equations, given a sequence of shocks $\{\hat{\xi}_t, \hat{\varepsilon}_t^i, \hat{M}_t, \hat{\lambda}_{w,t}\}$ and the actual level of productivity at date t, A_t^{data}:

$$A_{t+1}^{f,-\infty} = A_t^{data}(\hat{g}_{t+1}^f + \log(1+g_{ss}))$$

$$Y_t^{f,-\infty} = A_t^{data}(\hat{y}_t^f + \log y_{ss})$$

$$C_t^{f,-\infty} = A_t^{data}(\hat{c}_t^f + \log c_{ss})$$

Appendix D.9.3. sticky-wage allocation

Definition Appendix D.6 (sticky-wage allocation)**.** The *sticky-wage allocation* is defined as sequence of variables $\{Y_t,\ A_{t+1},\ C_t,\ \hat{\pi}_t^w,\ \hat{c}_t,\ \hat{y}_t,\ \hat{g}_{t+1},\ \hat{i}_t,\ \hat{L}_t,\ \hat{w}_t,\ \hat{\pi}_t,\ \hat{V}_t\}$ which satisfy the equations A.1- A.9 and the following equations, given a sequence of shocks $\{\hat{\xi}_t, \hat{\varepsilon}_t^i, \hat{M}_t, \hat{\lambda}_{w,t}\}$ and initial level of productivity A_0:

$$A_{t+1} = A_t(\hat{g}_{t+1} + \log(1+g_{ss}))$$

$$Y_t = A_t(\hat{y}_t + \log y_{ss})$$

$$C_t = A_t(\hat{c}_t + \log c_{ss})$$

A_t^{data} corresponds to A_t defined under the sticky-wage allocation.

Appendix E. Proposition Proofs

Proposition (Proposition 1: Steady State Efficiency)**.** *Assuming the policy maker has access to non-distortionary lump-sum taxes, the steady state of the competitive equilibrium can be made efficient using the following three fiscal tools :*
a) sales subsidy $\tau^p = 1 - \frac{1}{\alpha}$
b) wage tax cut $\tau^w = \lambda_w$ *, and*
c) research tax /subsidy $\tau^r = 1 - \left[\left(\frac{\gamma l^*(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}}{1-\beta(1-z^*)}\right)\left(\frac{1-\beta}{(\gamma-1)c^*}\right)\right]$*, where terms with* $*$ *denote the efficient steady state values.*

Proof. Follows from Appendix Appendix D.6above. □

Proposition (Proposition 2)**.** *The (time-0) natural rate allocation coincides with the (time-0) first-best allocation under liquidity demand and monetary policy shocks.*

Proof. From Appendix C.9, the time-0 natural rate allocation under liquidity demand shocks and monetary policy shocks is characterized by:

$$\hat{y}_t^f = 0, \hat{c}_t^f = 0, \hat{g}_{t+1}^f = 0; \quad \forall t \geq 0$$

Because of the presence of time-varying taxes, the time-0 first-best allocation has the same solution for the corresponding variables $\{\hat{y}_t^*, \hat{c}_t^*, \hat{g}_{t+1}^*\}$. Hence, output at any time under (time-0) natural rate and (time-0) first-best allocations coincide (follows from the accounting identity eq E.3). Moreover, time-t natural rate and time-t first best allocations also coincide with each other. □

Proposition (Proposition 3). *Assume that the economy is at the efficient steady state at time $t = 0$, with given productivity level A_0. Under sticky wage allocation, quadratic approximation of representative agent's lifetime utility function $\mathbb{W}_0$ around the non-stochastic efficient steady state is given by*

$$\frac{\mathbb{W}_0 - \mathbb{W}_0^*}{U_{c_{ss}} y_{ss}}$$
$$= -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t \left[\lambda_y \underbrace{\left(\hat{y}_t - \frac{\beta}{1-\beta}\frac{1}{\nu + \frac{y}{c}}\hat{g}_{t+1}\right)^2}_{(i)} + \lambda_g \underbrace{\hat{g}_{t+1}^2}_{(ii)} + \lambda_\pi \underbrace{(\hat{\pi}_t^w)^2}_{(iii)}\right] + \mathcal{O}(||\hat{\xi}_t, \hat{\varepsilon}_t^i||^3) + t.i.p. \qquad \text{(E.1)}$$

(i) : labor efficiency gap, (ii): productivity growth rate gap, and (iii): wage inflation gap

where $\lambda_y = \left(\nu + \frac{y}{c}\right) > 0$, $\lambda_g = \frac{c}{y}\frac{\beta}{1-\beta}\left[\frac{\nu}{\nu+\frac{y}{c}}\frac{\beta}{1-\beta} + [(\varrho - 1)\eta_g + 1]\right] > 0$, $\lambda_\pi = \frac{1+\lambda_w}{\lambda_w}\frac{1}{\kappa_w} > 0$, $\kappa_w \equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\nu\left(1+\frac{1}{\lambda_w}\right))} > 0$, $\eta_g = \frac{1+g}{g} > 1$ *and t.i.p. stands for "terms independent of policy". $\mathbb{W}^*$ denotes welfare under the (time-0) first-best allocation. The approximation is scaled by the constant* $U_{c_{ss}} y_{ss} = \frac{y_{ss}}{c_{ss}}$ *(evaluated at the efficient steady state).*

Proof. The proof for this is detailed and builds on results shown above. First, note from Appendix D.4 that the solution of welfare function of the representative household is equivalent to the solution of the policy-relevant welfare function derived in Appendix D.2

We then derive a quadratic approximation of the policy-relevant lifetime welfare function. Since the problem is relatively complicated, we break the approximation into first solving for a setting with flexible wages. We show the derivation in the case of flexible wages, i.e. no pricing distortions, in Lemma 1 below. This simplifies the exposition. It is relatively standard to extend this proof to include nominal wage setting frictions. The extended proof is similar to the textbook proof of Galí (2015, Ch. 4) and is available on request. □

Lemma 1. *Quadratic approximation of $\boldsymbol{W}_t$ under flexible wages is given by*

$$-\frac{1}{2}\left[\lambda_y\left((\hat{y}_t-\hat{y}_t^*)-\frac{\beta}{1-\beta}\frac{1}{\nu+\frac{y}{c}}(\hat{g}_{t+1}-\hat{g}_{t+1}^*)\right)^2+\lambda_g(\hat{g}_{t+1}-\hat{g}_{t+1}^*)^2\right]+h.o.t.+t.i.p.$$

Proof. We will make use of following two approximation results as in Erceg Henderson Levin 2000:

$$\frac{dx}{x}\approx\hat{x}+\frac{1}{2}\hat{x}^2,\quad \hat{x}\equiv\ln x-\ln\bar{x}$$

If $x=\left[\int_0^1 x(j)^\phi dj\right]^{\frac{1}{\phi}}$, the logarithmic approximation of x is

$$\hat{x}\approx\int_0^1\hat{x}(j)dj+\frac{1}{2}\phi var_j\hat{x}(j)=\int_0^1\hat{x}(j)dj+\frac{1}{2}\phi\left[\int_0^1\hat{x}(j)^2dj-\left(\int_0^1\hat{x}(j)dj\right)^2\right]$$

Writing the per period utility as sum of three components:

$$W_t=u(c_t)-\int_0^1 v(L_t(h))dh+\frac{\beta}{1-\beta}w(g_{t+1})$$

At the Efficient Steady state,

$$\varrho\delta\frac{g^{\varrho-1}(1+g)}{c(\gamma-1)^\varrho}=\frac{\beta}{1-\beta}$$

$$y=\alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)L;\quad \omega=\frac{y}{cL^{1+\nu}}$$

$$c+\delta\left(\frac{g}{\gamma-1}\right)^\varrho=y$$

$$u_y=\frac{1}{c};u_g=\frac{-1}{c}\frac{\varrho\delta g^{\varrho-1}}{(\gamma-1)^\varrho};u_{yg}=\frac{1}{c^2}\frac{\varrho\delta g^{\varrho-1}}{(\gamma-1)^\varrho}$$

$$u_{yy}=-\frac{1}{c^2};u_{gg}=-\left[\left(\frac{\varrho\delta g^{\varrho-1}}{c(\gamma-1)^\varrho}\right)^2+\frac{\varrho(\varrho-1)\delta g^{\varrho-2}}{c(\gamma-1)^\varrho}\right]$$

$$v_y=\omega L^\nu=\frac{1}{c};v_{yy}=\frac{\omega\nu L^{1+\nu}}{y^2}=\frac{\nu}{yc}$$

$$w_g=\frac{\beta}{1-\beta}\frac{1}{1+g};w_{gg}=\frac{-\beta}{1-\beta}\frac{1}{(1+g)^2}$$

Second Order approximation of individual components of the welfare function is given by:

$$u_t=\bar{u}+yu_y\frac{dy_t}{y}+(1+g)u_g\frac{dg_{t+1}}{1+g}+y(1+g)\frac{dy_t}{y}\frac{dg_{t+1}}{1+g}+\frac{y^2}{2}u_{yy}\left(\frac{dy_t}{y}\right)^2+\frac{(1+g)^2}{2}u_{gg}\left(\frac{dg_{t+1}}{1+g}\right)^2+h.o.t.$$

$$v_t=\bar{v}+yv_y\frac{dy_t}{y}+\frac{y^2}{2}v_{yy}\left(\frac{dy_t}{y}\right)^2+h.o.t.$$

$$w_t = \bar{w} + (1+g)w_g \frac{dg_{t+1}}{1+g} + \frac{(1+g)^2}{2} w_{gg} \left(\frac{dg_{t+1}}{1+g}\right)^2 + h.o.t.$$

Using the Taylor approximation result that

$$\frac{dx}{x} = \hat{x} + \frac{1}{2}\hat{x}^2$$

where $\hat{x} = \log(x) - \log(x_{ss})$, we can write down the quadtraic approximation as :

$$u_t = yu_y \left[\hat{y}_t + \frac{1}{2}\hat{y}_t^2\right] + (1+g)u_g \left[\hat{g}_{t+1} + \frac{1}{2}\hat{g}_{t+1}^2\right] + y(1+g)\hat{y}_t\hat{g}_{t+1} + \frac{y^2}{2}u_{yy}\hat{y}_t^2 + \frac{(1+g)^2}{2}u_{gg}\hat{g}_{t+1}^2 + h.o.t. + t.i.p.$$

$$v_t = yv_y \left[\hat{y}_t + \frac{1}{2}\hat{y}_t^2\right] + \frac{y^2}{2}v_{yy}\hat{y}_t^2 + h.o.t. + t.i.p.$$

$$w_t = (1+g)w_g \left[\hat{g}_{t+1} + \frac{1}{2}\hat{g}_{t+1}^2\right] + \frac{(1+g)^2}{2}w_{gg}\hat{g}_{t+1}^2 + h.o.t. + t.i.p.$$

where $\hat{y}_t = \log y_t - \log y$, and $\hat{g}_{t+1} = \log(1+g_{t+1}) - \log(1+g)$.

Combining the three components, per period welfare function can be expressed as:

$$\begin{aligned}
W_t = & \left[yu_y - yv_y\right]\hat{y}_t + \left[(1+g)u_g + (1+g)w_g\right]\hat{g}_{t+1} + y(1+g)u_{yg}\hat{y}_t\hat{g}_{t+1} \\
& + \frac{1}{2}\left[yu_y + y^2u_{yy} - yv_y - y^2v_{yy}\right]\hat{y}_t^2 \\
& + \frac{1}{2}\left[(1+g)u_g + (1+g)^2u_{gg} + (1+g)w_g + (1+g)^2w_{gg}\right]\hat{g}_{t+1}^2 \\
& + h.o.t. + t.i.p.
\end{aligned}$$

note that following relations hold true at the efficient steady state

$$yu_y = yv_y; \quad (1+g)u_g + (1+g)w_g = 0; \quad y(1+g)u_{yg} = \frac{y}{c}\frac{\beta}{1-\beta}$$

$$yu_y + y^2u_{yy} - yv_y - y^2v_{yy} = -\frac{y}{c}\left[\frac{y}{c} + \nu\right]$$

$$(1+g)u_g + (1+g)^2u_{gg} + (1+g)w_g + (1+g)^2w_{gg} = -\left[\frac{\beta}{1-\beta} + \left(\frac{\beta}{1-\beta}\right)^2 + \frac{\varrho(\varrho-1)\delta(1+g)^2g^{\varrho-2}}{c(\gamma-1)^\varrho}\right]$$

Using these into the quadratic approximation of W_t and completing the squares we get

$$\begin{aligned}
W_t = & -\frac{1}{2}\frac{y}{c}\left(\nu + \frac{y}{c}\right)\left[(\hat{y}_t - \hat{y}_t^*) - \frac{\beta}{1-\beta}\frac{1}{\nu + \frac{y}{c}}(\hat{g}_{t+1} - \hat{g}_{t+1}^*)\right]^2 \\
& - \frac{1}{2}\frac{\beta}{1-\beta}\left[\frac{\nu}{\nu + \frac{y}{c}}\frac{\beta}{1-\beta} + \left[(\varrho-1)\eta_g + 1\right]\right](\hat{g}_{t+1} - \hat{g}_{t+1}^*)^2 + h.o.t. + t.i.p.
\end{aligned}$$

the term in the first bracket is the labor wedge.

□

Lemma 2. *Labor Wedge is given by*

$$\nu(\hat{L}_t - \hat{L}_t^*) + (\hat{c}_t - \hat{c}_t^*) - (\hat{w}_t - \hat{w}_t^*) = (\hat{y}_t - \hat{y}_t^*) - \frac{\beta}{1-\beta}\frac{1}{\nu + \frac{y}{c}}(\hat{g}_{t+1} - \hat{g}_{t+1}^*)$$

Proof. Use equations A.4, A.5 and A.7 from definition A.3 to substitute for $\hat{L}_t$, $\hat{c}_t$ and $\hat{w}_t$. Finally note that under efficient allocation, the labor wedge is zero, that is, $\nu\hat{L}_t^* + \hat{c}_t^* - \hat{w}_t^* = 0$. □

In the following Corollary, we show the conditions under which the welfare loss resulting from these productivity growth rate deviations is larger than that arising due to changes in the labor efficiency gap. We provide a sufficient condition for the growth rate gap to be of higher importance for stabilization than the labor efficiency wedge. We argue below that this condition is likely to be satisfied even for extreme values of parameters considered in the literature. This highlights the importance of stabilizing the productivity growth rate around the first-best allocation.

Corollary (Corollary: Importance of Growth Stabilization)**.** *The relative weight on growth rate gap is higher than the relative weight on labor efficiency wedge if*

$$\frac{\beta}{1-\beta} > \frac{y}{c}\left(\nu + \frac{y}{c}\right) \tag{E.2}$$

Proof. If $\frac{\beta}{1-\beta} > \frac{y}{c}\left(\nu + \frac{y}{c}\right)$, then it follows directly that :

$$\frac{\beta}{1-\beta}\left[\frac{\nu}{\nu + \frac{y}{c}}\frac{\beta}{1-\beta} + (\varrho - 1)\eta_g + 1\right] > \frac{y}{c}\left(\nu + \frac{y}{c}\right)$$

since all the terms in the square bracket on the LHS are positive and add to more than 1. □

Common calibration values of discount rate β at quarterly frequency lie in the range of $[0.98, 1)$. This implies a lower bound on the left hand side of the condition (E.2) at 49. We bound the right hand side as follows: consumption to output ratio in the US has fluctuated between 0.54 and 0.66 from 1960 -2014 (BEA). In fact, in the benchmark calibration of the model's efficient steady state, the consumption-output ratio is 0.88. It is higher than the data equivalent since we do not have physical capital investment or government spending.[†] Estimates of Frisch elasticity of labor $1/\eta$ in the micro literature lie between 0.1 and 0.5 (Chetty et al. 2016) while the macro literature uses the estimates in the range of (2,4). Using value of 0.1 for η^{-1} and very conservative estimate of 0.54 for c/y ratio, this implies an upper bound on the right hand side at 22. Hence for a wide range of parameter estimates used in the macroeconomics literature, the welfare loss from

[†]This model implied ratio is what is likely to be of consequence in extended models featuring capital and government spending. We thank an anonymous referee for bringing this point to our attention.

a given growth rate deviation is higher than the welfare loss from a similar change in labor efficiency gap. Intuitively, a given deviation in growth rate from steady state has long run, potentially permanent effects. On the other hand, fluctuations in the labor efficiency pertain to welfare losses only in the period these are encountered.

Proposition (Proposition 4: Optimal Policy away from ZLB)**.** *Given a process for liquidity demand and monetary policy shocks, optimal policy under sticky wage allocation tracks the natural rate of interest when the Zero Lower Bound constraint is slack.*

Proof. When the nominal interest rate is set equal to the natural interest rate (and is non-negative), the unique solution to the competitive equilibrium is

$$\hat{y}_t = 0; \quad \hat{c}_t = 0; \quad \hat{\pi}^w_t = 0; \quad \hat{g}_{t+1} = 0$$

which corresponds to the first-best allocation as shown in proof of Proposition 4. □

Corollary (Corollary 1)**.** *When the ZLB is slack, the time series of output under optimal policy is a trend stationary process (integrated of order zero), that is,*

$$\log Y_t = a + b * t$$

where $a = \log Y_0$ is the initial level of output, and $b = \log(1 + g_{ss})$ is the steady state productivity growth rate.

Proof. Under optimal policy, the productivity growth rate does not deviate from the steady state growth rate. Hence the series of output can be expressed as:

$$\log Y_T = \log Y_0 + \sum_{k=0}^{T-1}(1 + g_{ss}) = \log Y_0 + (T - 1)(1 + g_{ss}); \quad \forall t \geq 1$$

□

Proposition (Proposition 5: Output hysteresis)**.** *Given the monetary policy rule (eq 7) and in the absence of a zero lower bound constraint on the nominal interest rate, transitory (modeled as AR(1) process) liquidity demand shocks or monetary policy shocks induce a permanent deviation in the time series of output from the counterfactual (flexible wage-) level of output if and only if monetary policy is not a strict targeting rule i.e.*

$$Y_T \neq Y^e_T \iff \{\phi_\pi, \phi_y > 0 : \phi_\pi \not\to \infty \cup \phi_y \not\to \infty\}$$

where $1 < T < \infty$ such that $y_T \equiv \frac{Y_T}{A_T} = y$ (steady state value).

Proof. We give the proof for liquidity demand shocks. The proof is identical for monetary policy shocks. Note that

$$Y_T^e = (1+g_{ss})^T A_0 y; \quad Y_T = \prod_{k=0}^{T-1}(1+g_k)A_0 y$$

Taking a log difference in the two series

$$\log Y_T - \log Y_T^e = \sum_{k=0}^{T-1} \hat{g}_{k+1} = \psi_g^\xi \sum_{k=0}^{T-1} \epsilon_k \tag{E.3}$$

where ψ_g^ξ is the coefficient derived in Appendix Appendix C above. For a given sequence of shocks that does not add to zero (which is the case with AR(1) process), the difference in the two series depends on ψ_g^ξ. This parameter is 0 if and only if monetary policy rule is either a strict inflation targeting ($\phi_\pi \to \infty$) or a strict employment targeting rule $\phi_y \to \infty$.

□

Proposition (Proposition 6: Output Hysteresis at the ZLB)**.** *Given the monetary policy rule (eq 7), a positive shock to liquidity demand such that the zero lower bound is binding for finite time T^e results in a permanent gap in output from the flexible wage counterfactual.*

Proof. A positive shock to the liquidity demand that induces the ZLB under the Taylor rule results in wage deflation and drop in output for the duration of ZLB.
Under Eggertsson and Woodford (2003) two-state Markov Chain assumption, the system at time $t < T^e$ is in state S (short run) and can be expressed as:

$$(1-\mu)\hat{c}_S = \mu\hat{\pi}_S^w + \hat{r}_S$$

$$(1-\beta\mu)\hat{\pi}_S^w = \kappa_w(\hat{c}_S + \nu\hat{y}_S)$$

$$[(\varrho-1)\eta_g + 1]\hat{g}_S = \mu\hat{V}_S + (1-\mu)\hat{c}_S$$

$$\frac{rd}{y}\varrho\eta_g\hat{g}_S = \hat{y}_S - \frac{c}{y}\hat{c}_S$$

$$\hat{V}_S = \frac{1}{1-\eta_V\mu}\left[\eta_V\hat{y}_S + \eta_q(1-\mu)\hat{c}_S - (\eta_z+\eta_q)\hat{g}_S\right]$$

We can solve the last three equations to find a relationship between c and y:

$$\hat{c}_S = \eta_C\hat{y}_S; \quad \eta_C \equiv \frac{\frac{1-\eta_V\mu}{\mu}\frac{(\varrho-1)\eta_g+1}{\frac{rd}{y}\varrho\eta_g} + \frac{\eta_z+\eta_q}{\frac{rd}{y}\varrho\eta_g} - \eta_Y}{\left[\frac{1-\eta_V\mu}{\mu}\frac{(\varrho-1)\eta_g+1}{\frac{rd}{y}\varrho\eta_g}\frac{c}{y} + (1-\mu)\right] + \frac{\eta_z+\eta_q}{\frac{rd}{y}\varrho\eta_g}\frac{c}{y} + \eta_q(1-\mu)} < 1$$

We can solve the system for $t < T^e$:

$$\hat{y}_t = \psi_y r_S^n < 0; \hat{\pi}_t^w = \psi_p r_S^n < 0; \hat{g}_t = \psi_g r_S^n < 0$$

where $\psi_y = \frac{(1-\beta\mu)\eta_C^{-1}}{(1-\beta\mu)(1-\mu)-\kappa_w(\nu+\eta_C)\mu} > 0$, $\psi_p = \frac{\kappa_w(\nu+\eta_C)}{1-\mu\beta}\psi_y > 0$, and $\psi_g = \frac{1-\frac{c}{y}\eta_C}{\frac{rd}{y}\varrho\eta_g}\psi_y > 0$. We assume (by A2 in the main text) the system is locally determinate around the state S equilibrium defined above. Therefore using the accounting identity eq E.3 derived in the proof of Proposition 1, we can derive:

$$\log Y_t - \log Y_t^e = \sum_{k=0}^{t-1} \hat{g}_{k+1} = (T^e - 1)\psi_g r_S^n < 0; \quad \forall\ t \geq T^e$$

This is the permanent output hysteresis in our framework following a ZLB episode. □

Appendix E.1. Optimal Policy at the Zero Lower Bound

Appendix E.1.1. Optimal Commitment Solution at the ZLB

$$\mathcal{L}_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \begin{cases} \frac{1}{2}\left[\lambda_1(\hat{y}_t - \tilde{\chi}\hat{g}_{t+1})^2 + \lambda_2 \hat{g}_{t+1}^2 + (\hat{\pi}_t^w)^2\right] \\ +\phi_{1t}\left[\hat{c}_t - \hat{c}_{t+1} - \hat{\pi}_{t+1}^w - \hat{r}_t^n\right] \\ +\phi_{2t}\left[\hat{\pi}_t^w - \beta\hat{\pi}_{t+1}^w - \kappa_w(\hat{c}_t + \nu\hat{y}_t)\right] \\ +\phi_{3t}\left[-(\mathbb{E}_t\hat{c}_{t+1} - \hat{c}_t + \hat{g}_{t+1}) + \mathbb{E}_t\hat{V}_{t+1} - (\varrho - 1)\eta_g\hat{g}_{t+1}\right] \\ +\phi_{4t}\left[\frac{c}{y}\hat{c}_t + \frac{rd}{y}\varrho\eta_g\hat{g}_{t+1} - \hat{y}_t\right] \\ +\phi_{5t}\left[-\hat{V}_t + \eta_y\hat{y}_t - \eta_z\hat{g}_{t+1} - \eta_q(\mathbb{E}_t\hat{c}_{t+1} - \hat{c}_t + \hat{g}_{t+1}) + \eta_V\mathbb{E}_t\hat{V}_{t+1}\right] \end{cases}$$

First Order conditions:

$$\phi_{1t} - \kappa_w\phi_{2t} + \phi_{3t} + \frac{c}{y}\phi_{4t} + \eta_q\phi_{5t} - \beta^{-1}\left[\phi_{1t-1} + \phi_{3t-1} + \eta_q\phi_{5t-1}\right] = 0$$

$$\lambda_1(\hat{y}_t - \tilde{\chi}\hat{g}_{t+1}) - \phi_{2t}\kappa_w\nu - \phi_{4t} + \phi_{5t}\eta_y = 0$$

$$-\lambda_1\tilde{\chi}(\hat{y}_t - \tilde{\chi}\hat{g}_{t+1}) + \lambda_2\hat{g}_{t+1} - [(\varrho - 1)\eta_g + 1]\phi_{3t} + \frac{rd}{y}\varrho\eta_g\phi_{4t} - (\eta_z + \eta_q)\phi_{5t} = 0$$

$$\hat{\pi}_t^w + \phi_{2t} - \phi_{2t-1} - \beta^{-1}\phi_{1t-1} = 0$$

$$-\phi_{5t} + \beta^{-1}\left[\phi_{3t-1} + \eta_V\phi_{5t-1}\right] = 0$$

$$\phi_{1t} \geq 0, \quad i_t \geq 0, \quad \phi_{it} i_t = 0$$

Appendix E.1.2. Optimal Discretionary Solution at the ZLB

Following is the Lagrangian for the Discretion policy

$$\mathcal{L}_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \begin{cases} \frac{1}{2}\left[\lambda_1(\hat{y}_t - \tilde{\chi}\hat{g}_{t+1})^2 + \lambda_2 \hat{g}_{t+1}^2 + (\hat{\pi}_t^w)^2\right] \\ +\phi_{1t}\left[\hat{c}_t - \hat{c}_{t+1}^e - \hat{\pi}_{t+1}^{we} - \hat{r}_t^n\right] \\ +\phi_{2t}\left[\hat{\pi}_t^w - \beta\hat{\pi}_{t+1}^{we} - \kappa_w(\hat{c}_t + \nu\hat{y}_t)\right] \\ +\phi_{3t}\left[-(\hat{c}_{t+1}^e - \hat{c}_t + \hat{g}_{t+1}) + \hat{V}_{t+1}^e - (\varrho - 1)\eta_g\hat{g}_{t+1}\right] \\ +\phi_{4t}\left[\frac{c}{y}\hat{c}_t + \frac{rd}{y}\varrho\eta_g\hat{g}_{t+1} - \hat{y}_t\right] \\ +\phi_{5t}\left[-\hat{V}_t + \eta_y\hat{y}_t - \eta_z\hat{g}_{t+1} - \eta_q(\hat{c}_{t+1}^e - \hat{c}_t + \hat{g}_{t+1}) + \eta_V\hat{V}_{t+1}^e\right] \end{cases}$$

$\lambda_1 = \kappa_w\left(\nu + \frac{y}{c}\right)\frac{\lambda_w}{1+\lambda_w}$, $\tilde{\chi} = \frac{\beta}{1-\beta}\frac{1}{\nu+\frac{y}{c}}$, and $\lambda_2 = \kappa_w\frac{c}{y}\frac{\beta}{1-\beta}\left[\frac{\nu}{\nu+\frac{y}{c}}\frac{\beta}{1-\beta} + [(\varrho-1)\eta_g + 1]\right]\frac{\lambda_w}{1+\lambda_w}$

First Order conditions:

$$\phi_{1t} - \kappa_w\phi_{2t} + \phi_{3t} + \frac{c}{y}\phi_{4t} + \eta_q\phi_{5t} = 0$$

$$\lambda_1(\hat{y}_t - \tilde{\chi}\hat{g}_{t+1}) - \phi_{2t}\kappa_w\nu - \phi_{4t} + \phi_{5t}\eta_y = 0$$

$$-\lambda_1\tilde{\chi}(\hat{y}_t - \tilde{\chi}\hat{g}_{t+1}) + \lambda_2\hat{g}_{t+1} - [(\varrho-1)\eta_g + 1]\phi_{3t} + \frac{rd}{y}\varrho\eta_g\phi_{4t} - (\eta_z + \eta_q)\phi_{5t} = 0$$

$$\hat{\pi}_t^w + \phi_{2t} = 0$$

$$\phi_{5t} = 0$$

$$\phi_{1t} \geq 0, \quad i_t \geq 0, \quad \phi_{it}i_t = 0$$

Proposition (Proposition 7: Optimal Discretionary Policy at the ZLB)**.** *If Assumptions A1 and A2 hold and for a given level of productivity at time 0, A_0, the Markov equilibrium is characterized by:*

$$\log A_1 = \log A_0 + \log(1 + g_{ss})$$

for $0 < t < T^e$

$$\hat{y}_t = \psi_y r_S^n < 0; \hat{\pi}_t^w = \psi_p r_S^n < 0; \hat{g}_t = \psi_g r_S^n < 0$$

$$\log A_{t+1} = \log A_t + \psi_g r_S^n$$

and when $t \geq T^e$

$$\hat{y}_t = \hat{\pi}_t^w = \hat{g}_t = 0$$

$$\log A_{t+1} = \log A_{t+1}^* + (T^e - 1)\psi_g r_S^n < \log A_{t+1}^*$$

where $\psi_y = \frac{1-\beta\mu}{(1-\beta\mu)(1-\mu)\eta_C - \kappa_w(\nu+\eta_C)\mu} > 0$*,* $\psi_p = \frac{\kappa_w(\nu+\eta_C)}{1-\mu\beta}\psi_y > 0$*, and* $\psi_g = \frac{1-\frac{c}{y}\eta_C}{\frac{rd}{y}\varrho\eta_g}\psi_y > 0$*.* A_{t+1}^* *is the (time-0) first-best output at time* $t+1$*.*

Proof. First note that the policymaker sets the policy rate to the unconstrained optimal policy rate as soon as the zero lower bound stops binding that is for $t \geq T^e$. The discretionary policy (MPE) taking into account the ZLB constraint is defined by the first order conditions derived above and the structural relations. The optimal policy when the ZLB stops binding involves setting ϕ_{it}, the Lagrange multiplier on the zero lower bound constraint, to 0. This reduces the system of equations to the familiar unconstrained policy of setting interest rate equal to the natural interest rate such that output and inflation are back to the (unconstrained) steady state. This constitutes a unique bounded solution and proves that there is no inertia in the discretionary policy. Remains to show that under the zlb, it is optimal to set interest rate to 0. Suppose it is not then, as discussed above, the Lagrange multiplier on ZLB constraint must be 0 and thus output and inflation must be at the steady state. But this leads to a violation of the AD equation, which is not satisfied. Next we solve for the values of endogenous variables . Under the assumed Eggertsson and Woodford two-state Markov Chain, the system at time $t < T^e$ is in state S (short run) and can be expressed as:

$$(1-\mu)\hat{c}_S = \mu\hat{\pi}^w_S + \hat{r}_S$$

$$(1-\beta\mu)\hat{\pi}^w_S = \kappa_w(\hat{c}_S + \nu\hat{y}_S)$$

$$[(\varrho - 1)\eta_g + 1]\hat{g}_S = \mu\hat{V}_S + (1-\mu)\hat{c}_S$$

$$\frac{rd}{y}\varrho\eta_g\hat{g}_S = \hat{y}_S - \frac{c}{y}\hat{c}_S$$

$$\hat{V}_S = \frac{1}{1-\eta_V\mu}\left[\eta_V\hat{y}_S + \eta_q(1-\mu)\hat{c}_S - (\eta_z+\eta_q)\hat{g}_S\right]$$

We can solve the last three equations to find a relationship between c and y:

$$\hat{c}_S = \eta_C\hat{y}_S; \quad \eta_C \equiv \frac{\frac{1-\eta_V\mu}{\mu}\frac{(\varrho-1)\eta_g+1}{\frac{rd}{y}\varrho\eta_g} + \frac{\eta_z+\eta_q}{\frac{rd}{Y}\varrho\eta_g} - \eta_Y}{\left[\frac{1-\eta_V\mu}{\mu}\frac{(\varrho-1)\eta_g+1}{\frac{rd}{y}\varrho\eta_g}\frac{c}{y} + (1-\mu)\right] + \frac{\eta_z+\eta_q}{\frac{rd}{y}\varrho\eta_g}\frac{c}{y} + \eta_q(1-\mu)} < 1$$

We can solve the system for $t < T^e$:

$$\hat{y}_t = \psi_y r^n_S < 0; \hat{\pi}^w_t = \psi_p r^n_S < 0; \hat{g}_t = \psi_g r^n_S < 0$$

where $\psi_y = \frac{(1-\beta\mu)\eta_C^{-1}}{(1-\beta\mu)(1-\mu)-\kappa_w(\nu+\eta_C)\mu\eta_C^{-1}} > 0$, $\psi_p = \frac{\kappa_w(\nu+\eta_C)}{1-\mu\beta}\psi_y > 0$, and $\psi_g = \frac{1-\frac{c}{y}\eta_C}{\frac{rd}{y}\varrho\eta_g}\psi_y > 0$. We assume (by A2 in the main text) the system is locally determinate around the state S equilibrium defined above. Therefore by the law of motion of productivity, we can derive that:

$$\log A_{t+1} = \log A_t + \psi_g r^n_S; \quad \forall 0 < t < T^e$$

Second part of the proposition (when $t \geq T^e$) follows from Proposition 6. □

Appendix E.2. consumption-equivalent welfare loss

We derive the consumption equivalent welfare loss relative to the (time-0) first best allocation as follows:

We discussed above in Appendix D.2 that the lifetime welfare function can be re-written as :

$$\mathbb{W}_0 = \sum_{s=0}^{\infty} \beta^s \left[\log c_s - v(L_s) + \frac{\beta}{1-\beta} \log(1 + g_{s+1}) \right]$$

Assuming a permanent gain in consumption $b \geq 0$ percent, the welfare at the efficient allocation is given by:

$$\mathbb{W}_0^*(b) = \sum_{s=0}^{\infty} \beta^s \left[\log(c_s(1+b)) - v(L_s) + \frac{\beta}{1-\beta} \log(1 + g_{s+1}) \right] = \mathbb{W}_0^*(b=0) + \frac{1}{1-\beta} \log(1+b)$$

Equating this to the welfare under the sticky wage allocation:

$$\mathbb{W}_0 = \mathbb{W}_0^*(b)$$
$$\Longleftrightarrow (1-\beta)(\mathbb{W}_0 - \mathbb{W}_0^*(b=0)) = \log(1+b) \approx b$$

Using the quadratic approximation derived above, we can solve for b.

Under monetary policy shocks, liquidity demand shocks and wage markup shocks, the first-best allocation corresponds with the no-fluctuations allocation. Hence the consumption-equivalent welfare loss is relative to the Balanced Growth Path. However under productivity shocks, the first-best allocation departs from the Balanced Growth path and the consumption equivalent welfare loss derived above is non-standard.

Appendix E.3. Simple targeting rules

We follow Chung et al. (2015) in implementing a simple version of operational rules. Simple nominal wage level targeting takes the form:

$$\hat{w}_t + \hat{y}_t - \hat{y}_t^f = 0,$$

where $w_t = \frac{W_t}{A_t}$ is the normalized wage level, $y_t = \frac{Y_t}{A_t}$ is normalized output, hats refer to log deviations from steady state and superscript f denotes the corresponding variable under flexible wage allocation. Simple hysteresis targeting rule is

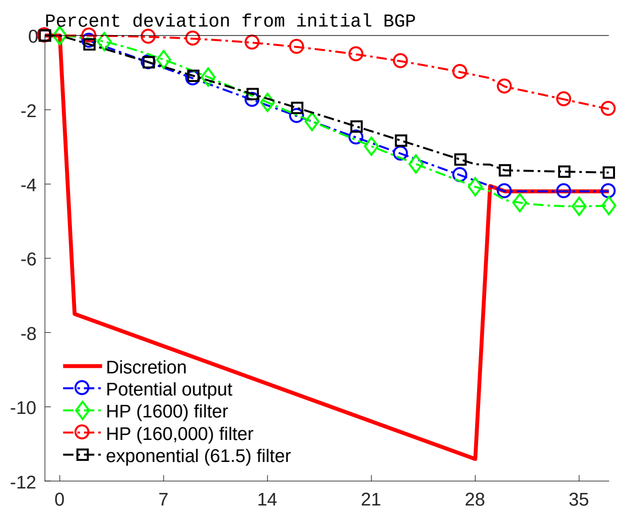
$$h_{t+1} + \hat{y}_t - \hat{y}_t^f = 0,$$

where h_{t+1} is the hysteresis term defined as a sum of productivity growth rate deviations (from steady state) because of the history of shocks realized until time t.

Appendix E.4. ideal statistical filters in the DSGE model

We follow Cúrdia, Ferrero, Ng and Tambalotti (2015) to implement statistical filters in the rational expectations model. Figure E.2 plots the potential output estimates from various (ideal) statistical filters against the time-t potential output and the actual output under a 28 period realization of binding ZLB from a two-state Markov chain. Output is plotted in percent deviations from the initial balanced growth path. The policymaker follows optimal discretionary (MPE) policy. We plot estimates from an exponential filter with smoothing parameter 61.5, and two HP filters with smoothing parameters 1600 and 160,000 respectively. The smoother trend from this higher value of λ in HP filter is closer to the time-0 potential output than the standard HP filter based trend with parameter set to1600.

Figure E.2: Real-time Estimate of Potential Output



Source: Authors' calculations.
Note: The figure reports one realization of output, potential output, two-sided HP filter, and exponential filter based output from a two-state Markov chain for the natural interest rate. In period 1, the natural interest rate becomes negative, stays there for 28 quarters, and then returns to its full employment steady state. The smoothing parameter was fixed at values of 1600 and 160,000 in the two HP filter implementations, and 61.5 in exponential filter. Output is reported in percent deviation from its pre-shock trend level.

A given stochastic process y_t is to be decomposed into a cyclical component and a trend component. The ideal filters construct a linear projection of the time-series to filter out cyclical components, which requires knowledge of spectral density of y_t. Cúrdia et al. (2015)'s insight was to use the model based rational expectation forecasts to expand the available sample (building on Christiano and Fitzgerald 2003). We briefly show the formulas used to construct the ideal filters in the DSGE model, and refer the reader to Cúrdia et al. (2015)'s appendix for details.

- **HP filter**: The ideal HP filter based gap (Baxter and King, 1999) with parameter λ is defined as

$$[1 + \lambda(1 - L)^2(1 - F)^2]x_t^{HP(\lambda)}] = \lambda(1 - L)^2(1 - F)^2 y_t,$$

with lags denoted with L and forwards operators with F. Under rational expectations, $Fy_t = E_t y_{t+1}$ can be constructed. λ is set to 1600 as per the standard convention with quarterly time-periods (King and Rebelo, 1993). We also show estimates from a high smoothing parameter 160,000 to illustrate a smoother trend.

- **exponential filter**: It is defined by

$$[1 + \lambda(1 - L)]x_t^{Exp} = \lambda(1 - L)y_t$$

λ is set to 61.5 as in Cúrdia et al. (2015) to match the gain of HP filter at frequency $\omega = 2\pi/32$, which corresponds to an eight year cycle (King and Rebelo, 1993).

Appendix F. Fiscal Policy Multipliers at the ZLB

We follow Eggertsson (2011) and investigate the fiscal multipliers in an environment with hysteresis. The key insight is that temporary targeted fiscal policy interventions have long-run implications. We show that R&D investment subsidies are expansionary; in related work, it has been shown that debt-financed fiscal policy can be self-financing in hysteresis-prone environments (see Eggertsson et al. (2016)). Similar results on paradox of toil, paradox of thrift, and expansionary government spending multipliers follow from our setup. We maintain our assumption of government's balanced budget.[†] Further we assume that following bounds on μ hold (as in Eggertsson (2011)):

$$\mu < \frac{((\varrho - 1)\eta_g + 1)(\gamma - 1)}{\beta\gamma}$$

$$(1 - \beta\mu)(1 - \mu) - \kappa_w(\nu + \eta_C)\mu\eta_C^{-1} > 0$$

Appendix F.1. R&D Investment Subsidy

Assume a temporary research subsidy is implemented $\hat{\tau}_S^r > 0$ for $S \in [1, T^e)$. Under Eggertsson and Woodford (2003)'s two-state Markov chain assumption, a competitive equilibrium of the model is given by:

for $t < T^e$:

$$\hat{y}_t = \psi_y r_S^n + \psi_\tau^y \hat{\tau}_S^r$$

$$\hat{\pi}_t^w = \psi_p r_S^n + \psi_\tau^p \hat{\tau}_S^r$$

$$\hat{g}_{t+1} = \psi_g r_S^n + \psi_\tau^g \hat{\tau}_S^r$$

[†]In our model, issuances of government bonds may be directly expansionary since bonds enter the utility function. For comparisons with earlier literature, we here work with government interventions that do not affect the welfare of the households directly (through bonds in utility for example).

where $\psi_y = \frac{(1-\beta\mu)\eta_C^{-1}}{(1-\beta\mu)(1-\mu)-\kappa_w(\nu+\eta_C)\mu\eta_C^{-1}} > 0$,
$\psi_\tau^y = \frac{1}{\frac{1-\eta_V\mu}{\mu}\frac{(\varrho-1)\eta_g+1}{\frac{R}{Y}\varrho\eta_g}+\frac{\eta_z+\eta_q}{\frac{rd}{y}\varrho\eta_g}-\eta_Y}\frac{(1-\beta\mu)(1-\mu)-\kappa_w\mu}{(1-\beta\mu)(1-\mu)-\kappa_w(\nu+\eta_C)\mu\eta_C^{-1}} > 0$
$\psi_p = \frac{\kappa_w(\nu+\eta_C)}{1-\mu\beta}\psi_y > 0$, and $\psi_g = \frac{1-\frac{c}{y}\eta_C}{\frac{rd}{y}\varrho\eta_g}\psi_y > 0$
$\psi_\tau^p = \frac{\kappa_w}{1-\mu\beta}\left[\frac{(1+\nu\eta_c^{-1})((1-\beta\mu)(1-\mu)-\kappa_w\mu)}{(1-\beta\mu)(1-\mu)-\kappa_w(\nu+\eta_C)\mu\eta_C^{-1}} - 1\right] > 0.$
$\psi_g = \frac{1-\frac{c}{y}\eta_C}{\frac{rd}{y}\varrho\eta_g}\psi_y > 0$
$\psi_\tau^g = \frac{1-\frac{c}{y}\eta_C}{\frac{rd}{y}\varrho\eta_g}\psi_\tau^y - \frac{\frac{c}{y}}{\frac{rd}{y}\varrho\eta_g}\eta_x = \frac{1}{\frac{rd}{y}\varrho\eta_g}\left[\psi_\tau^y - \frac{c}{y}\eta_x\left[\frac{((1-\beta\mu)(1-\mu)-\kappa_w\mu)}{(1-\beta\mu)(1-\mu)-\kappa_w(\nu+\eta_C)\mu\eta_C^{-1}} - 1\right]\right] > 0.$

Proof. The system of equations at time $t < T^e$ is in state S (short run) and can be expressed as:

$$(1-\mu)\hat{c}_S = \mu\hat{\pi}_S^w + \hat{r}_S$$

$$(1-\beta\mu)\hat{\pi}_S^w = \kappa_w(\hat{c}_S + \nu\hat{y}_S)$$

$$[(\varrho-1)\eta_g+1]\hat{g}_S - \hat{\tau}_S^r = \mu\hat{V}_S + (1-\mu)\hat{c}_S$$

$$\frac{rd}{y}\varrho\eta_g\hat{g}_S = \hat{y}_S - \frac{c}{y}\hat{c}_S$$

$$\hat{V}_S = \frac{1}{1-\eta_V\mu}\left[\eta_V\hat{y}_S + \eta_q(1-\mu)\hat{c}_S - (\eta_z+\eta_q)\hat{g}_S\right]$$

We can solve the last three equations to find a relationship between c and y:

$$\hat{c}_S = \eta_C\hat{y}_S - \eta_x\hat{\tau}_S^r; \quad \eta_C \equiv \frac{\frac{1-\eta_V\mu}{\mu}\frac{(\varrho-1)\eta_g+1}{\frac{rd}{y}\varrho\eta_g} + \frac{\eta_z+\eta_q}{\frac{rd}{y}\varrho\eta_g} - \eta_Y}{\left[\frac{1-\eta_V\mu}{\mu}\frac{(\varrho-1)\eta_g+1}{\frac{rd}{y}\varrho\eta_g}\frac{c}{y} + (1-\mu)\right] + \frac{\eta_z+\eta_q}{\frac{rd}{y}\varrho\eta_g}\frac{c}{y} + \eta_q(1-\mu)} < 1$$

$$\eta_x = \frac{1}{\left[\frac{1-\eta_V\mu}{\mu}\frac{(\varrho-1)\eta_g+1}{\frac{rd}{y}\varrho\eta_g}\frac{c}{y} + (1-\mu)\right] + \frac{\eta_z+\eta_q}{\frac{rd}{y}\varrho\eta_g}\frac{c}{y} + \eta_q(1-\mu)} > 0$$

Using this, the resulting AD-AS system can be expressed as:

$$(1-\mu)\eta_c\hat{y}_S = \mu\hat{\pi}_S^w + (1-\mu)\eta_x\hat{\tau}_S^r + \hat{r}_S$$

$$(1-\beta\mu)\hat{\pi}_S^w = \kappa_w(\eta_c+\nu)\hat{y}_S - \kappa_w\eta_x\hat{\tau}_S^r$$

Using these two equations, we can solve the model as above. □

The R&D subsidy is expansionary at the ZLB. R&D subsidy is analogous to the investment tax credit studied by Eggertsson (2011). Note that a supply side expansionary policy is contractionary at the ZLB if it reduces expectations of inflation. Here, this supply side policy increases the potential output of the economy without inducing the corresponding deflationary pressures. Instead the expectations of increased demand for research spending boosts inflation. Hence, a tax subsidy for non-tangible investment can be expansionary at the ZLB.

Furthermore, the long-run output is given by:

$$\log Y_{t+1} = \log Y^*_{t+1} + (T^e - 1)\psi_g r^n_S + (T^e - 1)\psi^g_\tau \hat{\tau}^r_S; \quad \forall t \geq T^e$$

The long-run output is higher by the increase in productivity growth rate achieved by higher research subsidies during the binding ZLB. Thus the long-run output multiplier for research subsidy is given by:

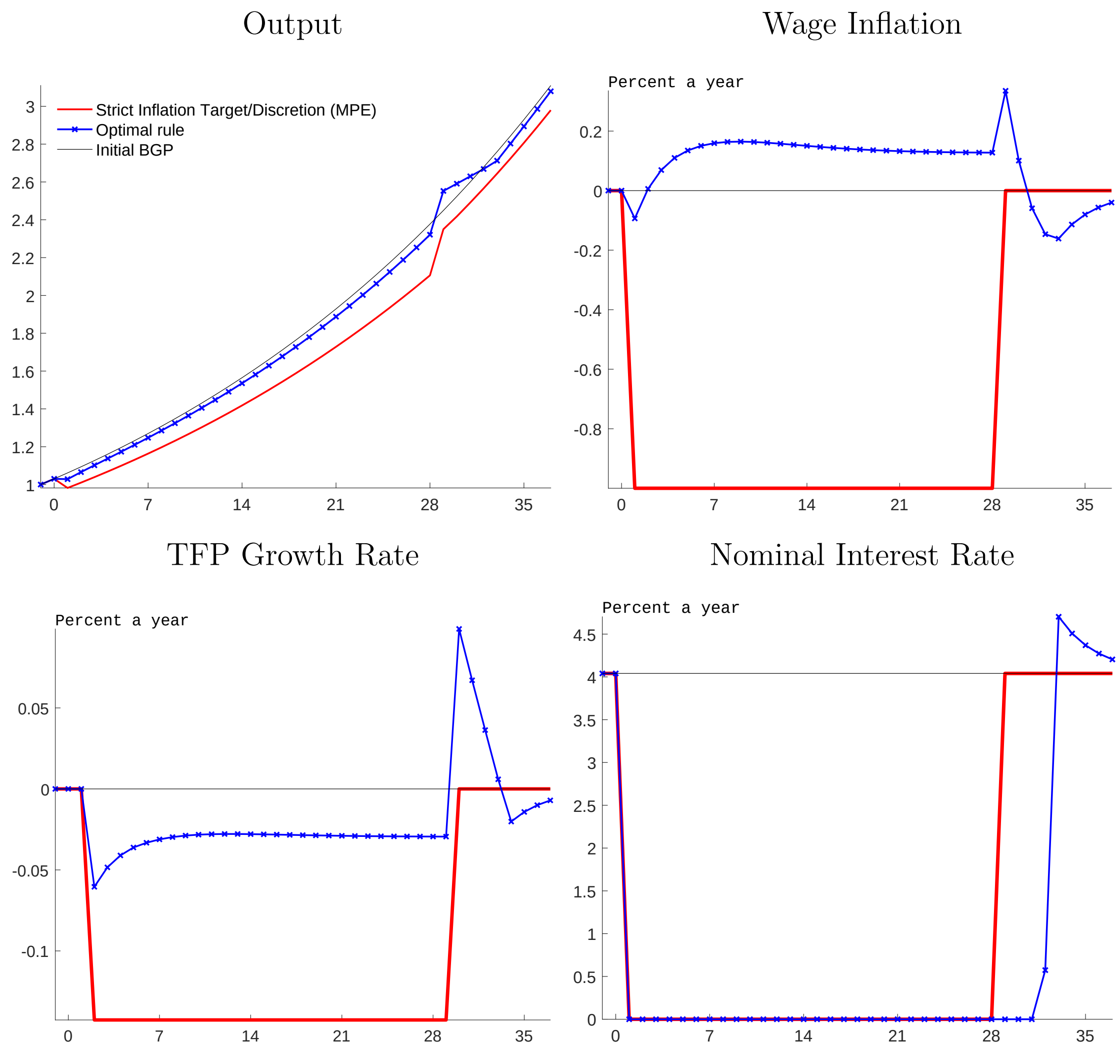
$$\frac{\partial Y_L}{\partial \hat{\tau}^r_S} = (T^e - 1)\psi^g_\tau > 0$$

In contrast to an exogenous TFP growth model, temporary R&D subsidies raise output permanently.[†]

Appendix G. Additional Figures

[†]We leave the analysis for various fiscal stabilization policies under endogenous growth (Denes et al., 2013; Mehrotra, 2018) for future work. See also Eggertsson and Garga (2019) for comparison of multipliers under sticky information and sticky prices assumptions.

Figure G.3: Optimal Policy at the Zero Lower Bound

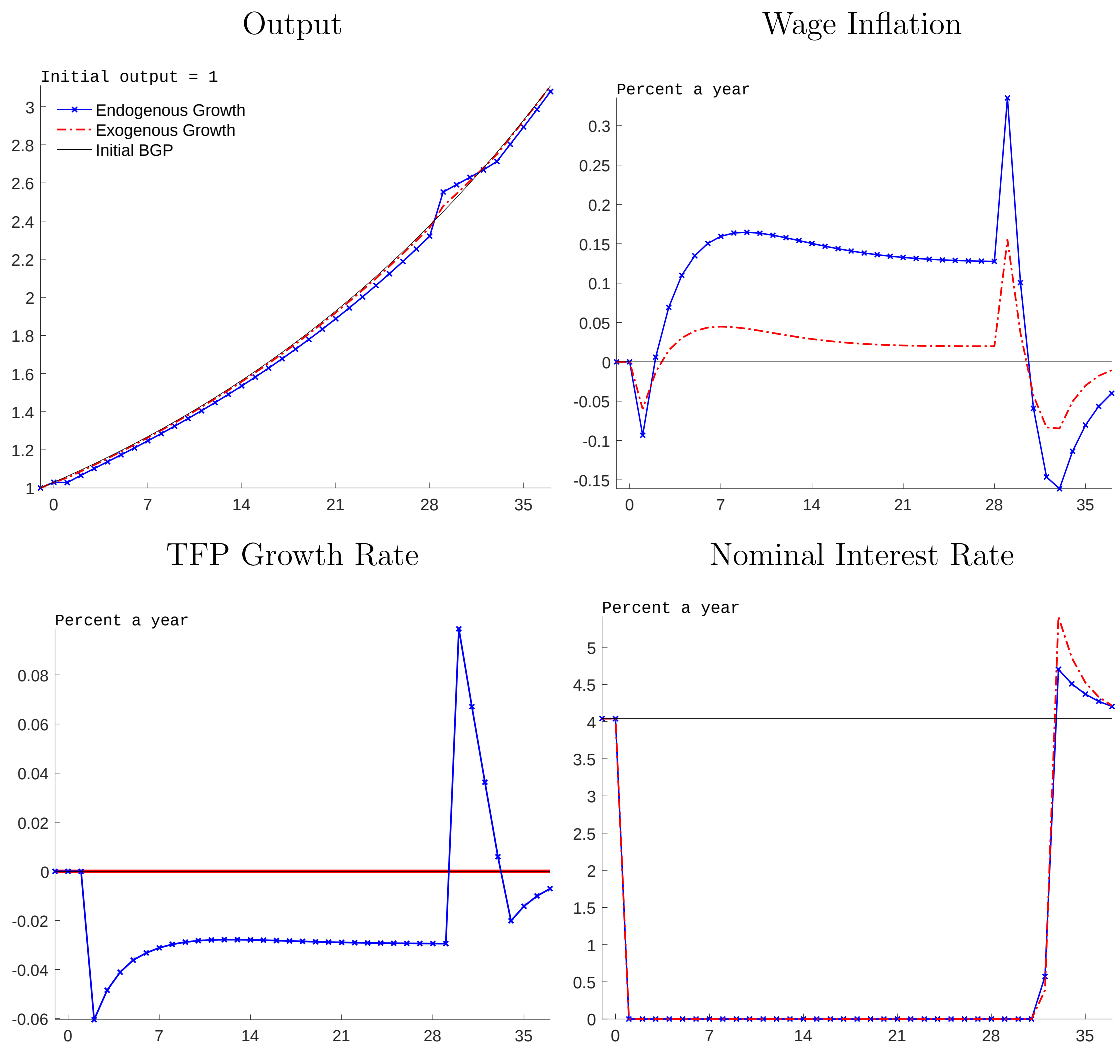


Source: Authors' calculations.
Note: The figure reports one realization of output, inflation, productivity growth rate and the nominal interest rate from a two-state Markov chain for the natural interest rate under alternate policy equilibria. In period 1, the natural interest rate becomes negative and stays there for 28 quarters, and returns to the full employment steady state. The realizations under a Taylor rule, Markov-Perfect Equilibrium (or discretionary) optimal policy, and optimal commitment policy are shown. TFP growth rate and wage inflation are plotted in (annualized) percent deviation from steady state. Output in period -1 is normalized at 1. Black line in the output graph plots evolution of deterministic trend at an annual 2% steady state growth rate.

Appendix H. Optimal policy under alternate shocks

In the main text, we focused on shocks such that the economy exhibits divine coincidence. The virtue of this exercise was that it did not matter whether output hysteresis was defined as deviation from the (time-0) first best, (time-0) natural rate or pre-recession trend output. In this section, we consider alternate demand and supply shocks, and analyze the optimal response of monetary policy in each case. The distinction between the three equilibrium concepts will become crucial now.

Figure G.4: Exogenous Productivity Comparison

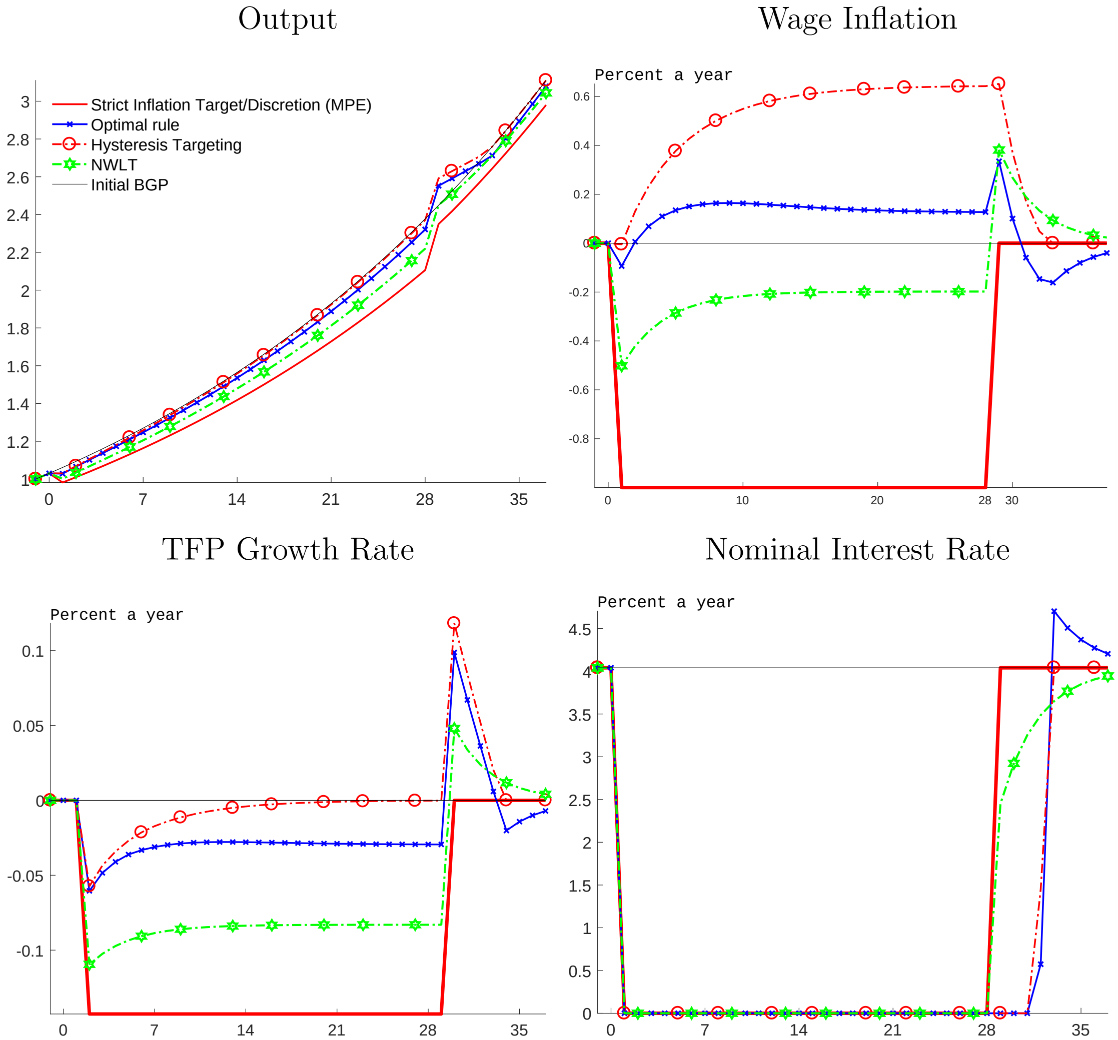


Source: Authors' calculations.
Note: The figure reports one realization of output, inflation, productivity growth rate and the nominal interest rate from a two-state Markov chain for the natural interest rate under alternate policy equilibria. In period 1, the natural interest rate becomes negative, stays there for 28 quarters, and returns back to the full employment steady state. Exogeneous growth denotes optimal policy in the exogenous growth benchmark (shutting down changes in R&D and TFP growth) from same steady state as the endogenous growth calibration. The optimal rule (dashed) denotes the optimal commitment equilibrium allocation with endogenous growth. FP growth rate and wage inflation are plotted in (annualized) percent deviation from the steady state. Output in period -1 is normalized at 1. The black line in the output graph plots the evolution of the deterministic trend at an annual 2% steady state growth rate.

Appendix H.1. Discount rate shocks

Discount rate shocks are modeled as shocks to household's discount rate. A positive shock to the discount rate temporarily makes the household more patient. This transmits to innovation through two opposing channels: One, lower discounting of future profits increases the present discounted value of innovation, thereby increasing investment in R&D. Two, in the presence of nominal rigidities, increased patience lowers aggregate consumption demand. If the aggregate demand channel is strong enough, output falls, thereby

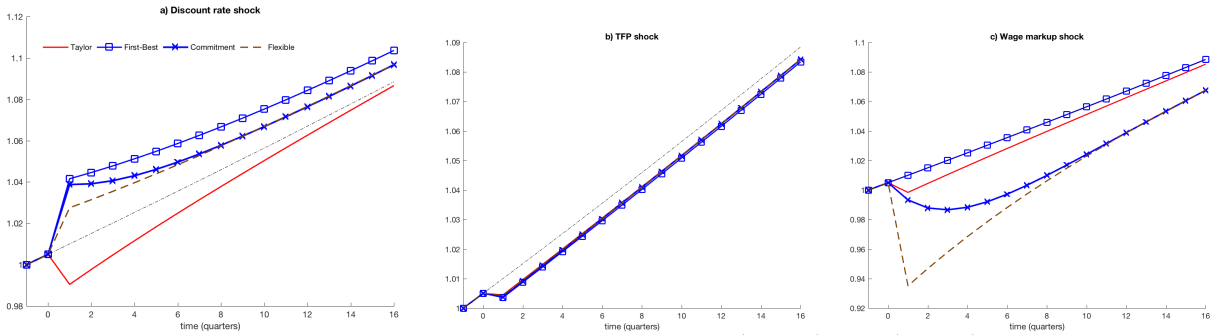
Figure G.5: Alternate Rules at the Zero Lower Bound



Source: Authors' calculations.
Note: The figure reports one realization of output, inflation, and the nominal interest rate from a two-state Markov chain for the natural interest rate under alternate policy equilibria. In period 1, the natural interest rate becomes negative and stays there for 28 quarters, and returns to the full employment steady state. The realizations under a Taylor rule, Markov-Perfect Equilibrium (or discretionary) optimal policy, optimal commitment policy, hysteresis targeting and nominal wage level targeting rule are shown. TFP growth rate and wage inflation are plotted in (annualized) percent deviation from steady state. Output in period −1 is normalized at 1. Black line in the output graph plots evolution of deterministic trend at an annual 2% steady state growth rate.

reducing the investment in R&D due to a shrunken market (*aggregate demand effect*). Under the first-best allocation, however, prices are flexible, so there is no negative aggregate demand channel. This leads to an increase in R&D relative to the pre-recession trend (figure H.6a, squared-blue graph). In the presence of nominal rigidities, however, the overall effect on R&D is determined by two opposing forces as described above. In our calibration, the aggregate demand channel dominates and investment in R&D and hence, TFP growth rate and output fall under a standard Taylor rule (figure H.6a, red graph).

Figure H.6: Path of GDP under TFP and wage markup shocks



Note: The figure reports model based evolution of GDP under discount rate (panel a), TFP (panel b) and wage markup shocks (panel c). Shocks are parametrized such that output falls by 1 percent on impact. For illustration, persistence of shocks is chosen to equal 0.9. Output in period -1 is normalized at 1. Black line plots evolution of deterministic trend at an annual 2% steady state growth rate.

The response of the first-best allocation and the flexible-wage allocation (figure H.6a, dashed-maroon graph) differ because of breakdown in divine coincidence under discount rate shocks. The entrepreneurs do not internalize the long-run benefits of innovation compared to the social planner despite the presence of an efficient steady state (Nuño, 2011). Replicating the flexible wage allocation is no longer an optimal policy. Infact, the natural rate of interest *r-star is an endogenous object* in this environment. Under optimal commitment equilibrium, the policy maker lowers the real rate in order to closely replicate the welfare gains under first-best allocation. This results in overshooting of output relative to both the flexible-price GDP and Taylor rule GDP (figure H.6a, crossed-blue graph).

Appendix H.2. Stationary TFP shocks

A negative productivity shock shrinks the resources available for consumption and R&D investment. It is optimal to reduce R&D investment in response to a temporary reduction in the level of total factor productivity. Temporarily lower productivity growth, as a result of low investment, cumulates to generate a permanent output gap relative to the pre-shock trend. Hence, the time-0 first best allocation features a unit-root process for output (figure H.6b, squared-blue graph). Since optimal monetary policy approximates the first-best allocation, the optimal commitment solution also admits output hysteresis (figure H.6b, crossed-blue graph).

Appendix H.3. Wage markup shocks

In the presence of cost-push shocks, the central bank faces a *tradeoff in stabilizing short-term inflation and long-run output.* The optimal commitment allocation (figure H.6c, crossed-blue graph) admits a permanent output gap. This result is a generalization of the short-run tradeoff in the exogenous growth new Keynesian model. With exogenous growth, the central bank counters a positive wage markup shock by committing to generating a negative output gap in the future. The same commitment under endogenous growth implies

Table H.1: Policy Rules : Welfare Comparison

Policy Rule	*Discount rate shock*	*Markup shock*	*Productivity Shock*	*Liq Demand Shock*	*MP shock*
Optimal rules					
Commitment	0.0018%	0.15%	0.00008%	0	0
Discretion	0.0035%	0.827%	0.0001%	0	0
Simple rules					
Taylor rule eq 7	0.0237%	2.21%	0.0003%	0.020%	0.025%
Hysteresis Targeting	0.0022%	5.32%	0.0013%	0	0
Wage Level Targeting	0.0022%	0.352%	0.00011%	0	0
Nominal GDP targeting	0.0022%	4.11%	0.0005%	0	0

Notes: Values report the conditional welfare loss starting from an efficient steady state. Welfare losses are computed as an average over 10,000 simulations, each starting at the same efficient steady state. Loss is expressed in consumption equivalent units (in percents of steady state consumption).

a reduction in market size for entrepreneurs and hence reduced incentive to undertake R&D. Thus, in a bid to reduce current wage inflation, the central bank keeps output permanently below the time-0 first best allocation. The inflation stabilization objective generates a long-run tradeoff for the central bank.[†]

Appendix H.4. Welfare analysis

In table H.1 we report the consumption equivalent welfare losses conditional on starting from an efficient steady state. These losses are computed as an average over 10,000 simulations with each starting at the same efficient steady state. Hysteresis targeting rule is of the form $h_{t+1} + y_t - y_t^f = 0$ rule, where superscript f denotes flexible wage allocation, h_t is (log) hysteresis determined at time $t-1$ and y_t is (log) stationarized output. Wage level targeting rule is implemented as $W_t + y_t - y_t^f = 0$, where W_t is the (log) nominal wage. Nominal GDP targeting takes the form: $P_t + h_{t+1} + y_t - y_t^f = 0$. In response to demand shocks, hysteresis targeting closely replicates the welfare achieved under optimal commitment. In response to supply shocks, it is an order of magnitude more costly (in terms of welfare) to implement hysteresis targeting relative to the optimal policy. Wage level targeting rule, which serves as the analogue of a price level targeting rule, is found to perform well in terms of welfare losses across all considered shocks. This highlights the importance of correctly identifying the source of business cycle fluctuations in the design of optimal monetary policy.

Appendix I. Quantitative Evaluation

So far, we advanced a channel for hysteresis by allowing monetary policy to have an effect on R&D investments and hence TFP growth. Second, we solved for optimal policy at ZLB assuming a liquidity demand

[†]Note that the time-0 first best allocation is a trend stationary process (figure H.6c, squared-blue graph). This is because we assume that the social planner has access to time-varying taxes to counter these shocks (Correia et al., 2013).

shock. Our analysis raises two questions: (i) does monetary policy influence productivity enhancing investments and the level of TFP in the data, and (ii) can a realistically calibrated liquidity demand shock generate a sizable recession. We answer both questions in the affirmative. We show empirical evidence consistent with key model predictions regarding monetary policy shocks. Contractionary monetary policy temporarily reduces R&D investment, firm entry, and has a persistent effect on TFP. Further, we conduct numerical exercises using a medium scale version of our model. A one time increase in liquidity demand, calibrated to match the increase in premium associated with very liquid assets during the financial crisis, can explain a third of the drop in output observed in the data during the Great Recession.[†]

Appendix I.1. Empirical Evidence

We estimate dynamic causal impacts of monetary policy on R&D investment, firm-entry and aggregate TFP. We interpret firm entry as an indicator for productivity enhancing investment for two reasons. First, we observe R&D investment for large firms in the data. These firms may not be significant drivers of TFP growth. Second, Decker et al. (2014), among others, have shown that firm entry is a significant driver of TFP growth. Consistent with the creative destruction literature, we interpret the number of innovating sectors in our model as counterpart of net firm entry in the data. The estimated impulse responses lend support for key predictions of our model: a contractionary monetary policy shock has a transitory negative effect on R&D investment and firm entry, and a persistent negative effect on TFP.

Empirical Strategy

Our empirical strategy is based on the recent literature (Jordà et al. 2019, and Ramey and Zubairy 2018) that combines the instrumental variables with the local projections (LP-IV) approach to directly estimate the structural IRFs. The series of (narratively- and high frequency-) identified monetary surprises e_t^m are treated as proxy for the true shocks ϵ_t^m. In the first-stage, we instrument a policy indicator (fed funds rate) with the relevant proxy.[†] In the second stage, we run a sequence of predictive regressions of the dependent variable on the instrumented policy indicator for different prediction horizons. The estimated sequence of regression coefficients of the instrumented policy indicator are then the impulse responses.

[†]Jordà et al. (2020) show evidence of long-run effects of monetary policy shocks using trilemma identification for seventeen advanced economies over 1890–2015. Palma (2019) finds persistent real effects of money injections through discoveries of precious metals from 16th to 18th century.

[†]The use of external instruments or proxy SVAR was developed by Stock (2008), and extended by Stock and Watson (2012) and Mertens and Ravn (2013). Gertler and Karadi (2015) combine high-frequency identification and proxy SVARs to estimate monetary policy impulse responses. Stock and Watson (2017) discuss connections between proxy SVAR and LP-IV approaches.

More specifically, we estimate the following second-stage LP specification for horizons h ∈ 0,..., H:

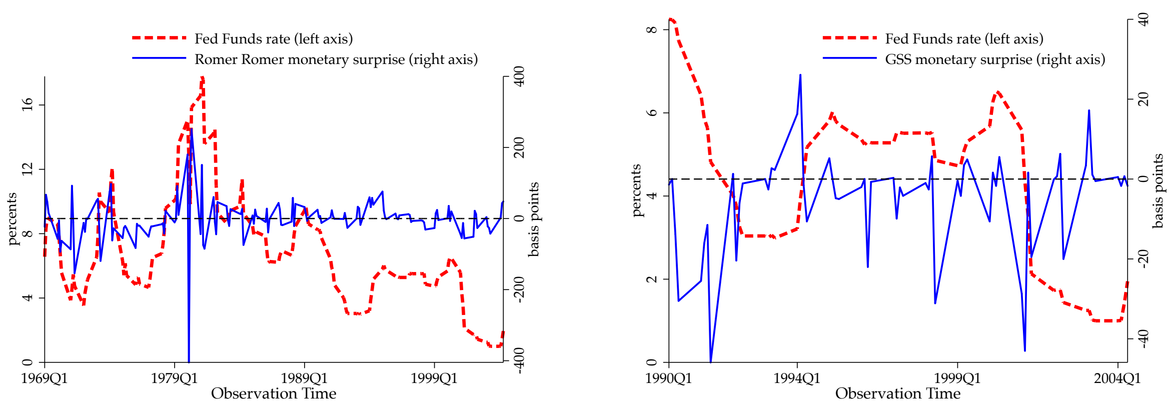
$$y_{t+h} = \alpha^h + \beta^h f\hat{f}r_t + \sum_p \theta^{ph} Z_{t-p} + \nu_{t+h} \tag{I.1}$$

$f\hat{f}r_t$ is the predicted policy instrument from the first-stage regression using identified monetary policy instruments e_t^m. The set Z_t includes lags of dependent variable, the policy indicator, the policy instrument, and the current and lagged conditioning variables that identify exogenous fluctuations in the monetary policy instrument and improve precision of standard errors (see Stock and Watson 2017). The conditioning variables are log real GDP and log GDP deflator. The dynamic coefficients of interest are, therefore, the estimates of β^h for $h = 0, 1, ..., H$. We compute standard errors based on heteroskedasticity and autocorrelation robust covariance matrix (Newey-West) estimators. The impulse responses for R&D investment at the firm-level are estimated in a similar manner, by conditioning on time-invariant firm-fixed effects, an aggregate time trend as well as two lags of time-varying firm-level controls (assets, cash holdings, short-term debt, and annual employment). The standard errors, in this case, are clustered at the firm-level.

Data: Instruments and Variables of Interest

We obtain two sequences of monetary policy surprises identified in the empirical literature. One is

Figure I.7: Policy Indicator and Monetary Policy Surprises



Note: The figure plots the Federal Funds rate against the monetary surprises. Two measures of monetary surprises are used in the main text. On the left, we plot the Romer & Romer (2004) narrative-identified monetary policy instruments. On the right, we plot the changes in current-month federal funds rate futures in a narrow 30 minute window around FOMC meeting announcements. These daily indicators are aggregated to the monthly frequency by adjusting for number of days left in the month. Monthly monetary surprises are summed to get the quarterly frequency aggregates. We take the FOMC days' announcement surprises from Gürkaynak et al. (2005)

narratively-identified series from Romer and Romer (2004) (RR). They decompose changes in the intended federal funds rate at the FOMC meetings into a *systematic* and a *residual shock* component. The residual shock is extracted from unexplained variation in a regression of target funds rate changes on changes in Greenbook forecasts of inflation, output growth and unemployment. The original monthly series from 1969-1996 has been recently extended by Wieland and Yang (2016) until 2007. The second set of surprises are measured using high-frequency data on the federal funds futures contracts. The rates on these contracts reflect market expectations of the average federal funds rate during that month. To identify the exogenous part of announced changes in monetary policy, Gürkaynak et al. (2005) (GSS) calculate changes in the traded rate in a narrow 30 minutes window around the FOMC press releases. We obtain this series for 1990-2007 by combining the data from GSS with that extended by Gorodnichenko and Weber (2016) and refer to these as HFI (high frequency instruments). An unweighted sum of these series is used to convert monthly into quarterly frequency. Figure I.7 plots series of obtained shocks against the effective federal funds rate. We use information on surprises until 2007Q4, before the financial crisis.[†]

As measures for R&D investment, we use two quarterly data series (denoting sample lengths used in parentheses): (i) log R&D investment deflated by GDP deflator available from NIPA (1969-2007), and (ii) firm-level R&D investment constructed from COMPUSTAT database (1990-2007). The construction of firm-level R&D investment data is described in the Appendix and follows the methodology common in the literature (Brown, Fazzari, and Petersen 2009, Terry 2017).[†] As measures of firm entry, we obtain two aggregate data series: (1) log number of business incorporations, and (2) (net) establishment births/(establishment births + establishment deaths). The first series is aggregated to quarterly level from a monthly Survey of Current Business produced until 1994 run by the Bureau of Labor and Statistics (BLS). The second series comes from a quarterly National Private Sector Business Employment Dynamics Data of BLS available 1993 onwards. Finally, log utilization-adjusted TFP and non-adjusted TFP measures are constructed by cumulating the respective TFP growth rate series obtained from (Fernald, 2014) over 1969-2007.

Results

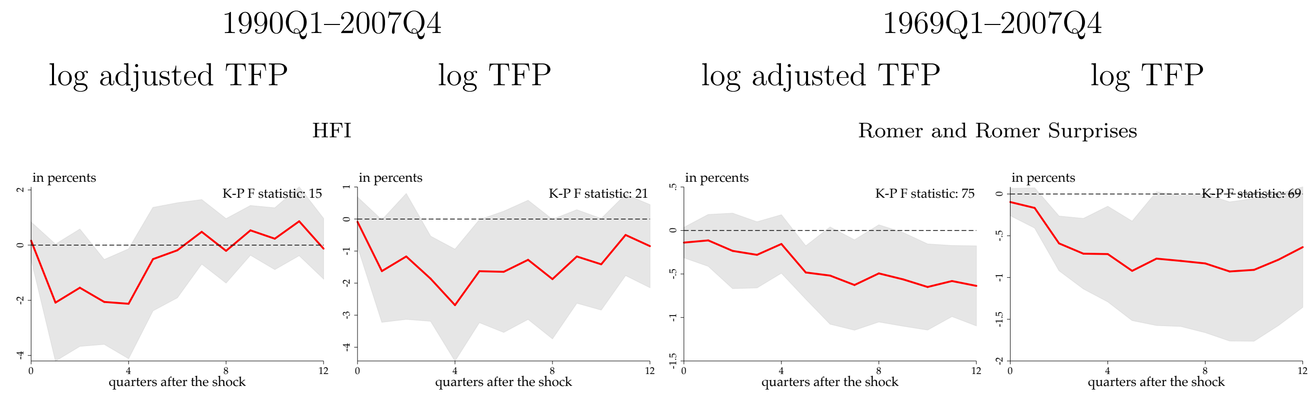
Figures I.8 and I.9 report our main empirical results using the HFI and RR instruments over different sample lengths. We report deviations from a constant trend following a 100 bps increase in federal funds rate. The shaded areas represent the 95% confidence intervals. We report the F-statistics for respective IRFs in the figures to verify instrument relevance. In most cases, the F statistic is above 23, a threshold for ten percent level constructed by Montiel-Olea and Pflueger (2013). Because of the shorter sample length, the HFI instrument does suffer from the weak-instruments issue.

In figure I.8, we plot the IRFs for utilization-adjusted TFP and raw TFP. Consistent with the dynamics of the model, the utilization-adjusted TFP declines gradually after a monetary policy shock. The IRFs for raw TFP decline by more than the fall in adjusted TFP because of higher fluctuations in factor utilizations induced by monetary policy shocks. The leveling off of the decline in raw TFP is consistent with the

[†]We exclude the rate cut of September 2001, to avoid the noise in the rates caused by the terrorist attacks.

[†]To provide a broad picture, the firm-level R&D sample data in year 2000 contained 3441 firms for which R&D investment information was available. These firms collectively employed 9.7% of total US Employment (Fred code: PAYEMS), spent 86% of total private R&D measured by NIPA and had sales worth 26% of US nominal GDP. Data construction discussed in Appendix I.2.

Figure I.8: Response of utilization adjusted TFP and TFP to 100 bps increase in Federal Funds Rate



Notes: The figure plots the estimated impulse response functions for log utilization adjusted TFP and non-adjusted TFP. Time is in quarters. Sample length, and instrument used are denoted on top of the figures. IRFs are computed using a local-projections IV approach. Current and two past-lagged values of log real GDP and inflation rate are used as conditioning variables. Regressions also include past values of the proxy, the federal funds rate, and the dependent variable. Kleibergen-Paap F statistic for weak instruments are reported in the figures. The standard errors are calculated using HAR-Newey-West standard errors. The shaded areas denote 95% confidence intervals.

persistent decline in adjusted TFP. This decline in TFP reaches -0.6% after 12 quarters (estimated on data from 1969-2007).

In Figure I.9, we plot the response of the number of new incorporations, establishment births, aggregate R&D and corporate R&D investments. Contractionary monetary policy shocks have a negative effect on these indicators. There is a delayed negative effect on R&D investment, which is not statistically significant for aggregate R&D but is statistically significant at the firm-level. Our benchmark model does not feature adjustment costs or frictions in R&D investment. As a result, the benchmark model exhibited a linear response of R&D investment to monetary policy shocks. In the medium scale model, we introduce adjustment costs in order to generate the curvature in the R&D response. The empirical findings align with the key predictions of our model: monetary policy influences long-run level of TFP. We next use these empirical findings to assess the quantitative relevance of our model.

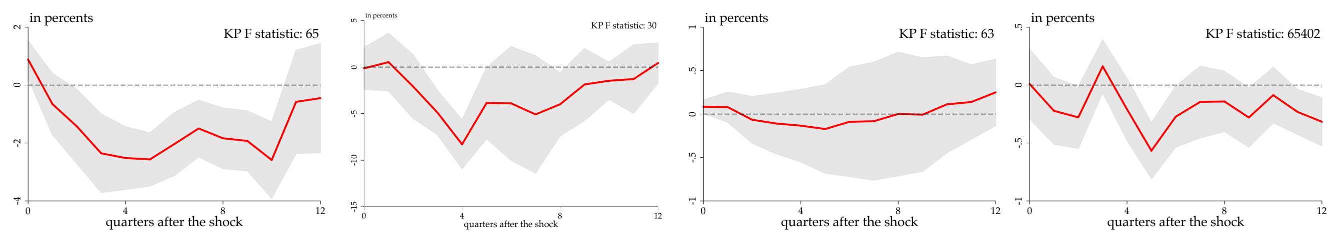
Appendix I.2. Additional details on data

Appendix I.2.1. Sources

- *Real GDP, GDP deflator, unemployment rate, R&D Investment* (1969 - present): St. Louis FRED database
- *TFP* (Quarterly, 1969 - present): Fernald (2014). We constructed the annualized TFP growth rates into a log TFP series.
- *Number of new business incorporations* (Monthly, 1969 - 1994): Survey of Current Business Jan-

Figure I.9: Response of Firm Entry, Aggregate R&D and Firm-level R&D to 100 bps increase in Federal Funds Rate

Firm Entry		R&D	
1969Q1–1994Q4	1993Q2–2007Q4	1969Q1–2007Q4	1990Q1–2007Q4
(log) no. of new incorp.	Net Establishment Births	Aggregate R&D	Firm-level R&D
Romer and Romer Surprises	HFI	Romer and Romer Surprises	HFI



Notes: The figure plots the estimated impulse response functions for firm entry, aggregate R&D and firm-level R&D. Two indicators for firm-level R&D are used: (1) log number of new incorporations available over 1969–1994, and (2) log number of net establishment births available since 1993Q2. Time is in quarters. Sample length, and instrument used are denoted above each figure. IRFs are computed using a local-projections IV approach. Current and two past-lagged values of log real GDP and inflation rate are used as conditioning variables. Regressions also include past values of the proxy, the federal funds rate, and the dependent variable. Kleibergen-Paap F statistic for weak instruments are reported in the figures. Firm -level R&D regressions also include two lags of assets, short debt, cash, employment, and firm-fixed effects. The standard errors are robust clustered at the firm-level. The shaded areas denote 95% confidence intervals.

uary/February 1996 supplement titled "Sources for Business Cycle Indicators" (discontinued) from the BEA website

- *Quarterly net establishment births* (Quarterly, 1993Q1 - present): National Private Sector Business Employment Dynamics Data, BLS
- *R&D Compustat* (Monthly, 1969 - 2010): Quarterly and Annual COMPUSTAT database from WRDS, (Quarterly, 1990Q1 - present)
- *Romer Romer shocks*: Romer and Romer (2004), Wieland & Yang (2016)
- *High frequency shocks* (Monthly, 1990 - 2010): Gorodnichenko & Weber (2016) and Gurkayanak, Sack & Swanson (2006).

Appendix I.2.2. Firm level R&D data construction

We downloaded COMPUSTAT data from the US Fundamentals Quarterly file available through Wharton Research Data Services (WRDS). Annual employment data came from the US Fundamentals Annual file. We follow Terry (2017) and make the following sample restrictions:

- Nonmissing total assets atq, SIC code sic, book value of capital ppentq, GAAP earnings ibq, operating earnings before depreciation EBITDA oibdpq, total sales saleq, value of equity ceqq
- Positive levels of assets and book value of capital: atq, ppentq > 0

- No utilities or financial firms as classified by SIC code: sic not in 6000's or 4900's

In the baseline regression at horizon 0, the sample included 4271 unique gvkey and 90385 firm-quarter observations between 1992Q1 and 2007Q4. Nominal variables were deflated using the GDP deflator. R&D investment is defined as the difference between log R&D stock in two consecutive periods. Following Brown, Fazzari & Petersen (2009), and Kabuckuoglu (2014), we construct R&D stock using perpetual inventory method as follows:

$$\mathrm{RD}^{stock}_{i,t} = (1-\delta^R)\mathrm{RD}^{stock}_{i,t-1} + \mathrm{XRDQ}_{i,t}$$

where $\mathrm{XRDQ}_{i,t}$ represents the real R&D expenditures of firm i at time t; δ^R is the depreciation rate . We assume $\delta^R = 15\%$(annualized), standard practice in the innovation literature.. Initial period R&D stock is assumed to be $\frac{\mathrm{XRDQ}_{i,0}}{\delta}$, where $\mathrm{XRDQ}_{i,0}$ is the first observation of R&D expenditures for firm i. We define R&D investment as:

$$\Delta\mathrm{R\&D}_{i,t} = \log \mathrm{RD}^{stock}_{i,t+1} - \log \mathrm{RD}^{stock}_{i,t}$$

Appendix I.3. Medium Scale DSGE Model with Schumpeterian growth

Appendix I.3.1. Model

For brevity, we sketch the additional features introduced into the benchmark model and leave the detailed model discussion to Appendix J. Capital is introduced in the production of intermediate good, following Howitt and Aghion (1998). Households own and accumulate capital subject to investment adjustment costs and rent it out to the intermediate good monopolists. The specification for investment adjustment costs follows the new Keynesian literature (Christiano et al., 2005). We append price-rigidity by introducing a retail sector that sells the final good produced by the perfectly competitive producer. Monopolistically competitive retailers set prices on a staggered basis following Calvo (1983). Further, we allow for variable capital utilization, and (internal) habits in consumption. Relative to the existing new Keynesian literature, we introduce adjustment costs in R&D expenditure. A particular functional form we use is $S^{rd} = \frac{\varkappa}{2}\left(\frac{R_t}{(1+g_{ss})R_{t-1}} - 1\right)^2$, which the entrepreneur takes as given while making her R&D investment decision. This feature helps the model match the curvature in R&D responses that is found in empirical IRFs (see Figure I.9 discussed above, as well as Moran and Queraltó 2018).

Appendix I.3.2. Calibration

We calibrate the model at a quarterly frequency. Table I.2 reports the calibrated values of parameters, that we discuss next:

Steady State Parameters

Steady state labor supply is normalized to 1. Six parameters are set to match six steady state targets. Table

I.3 reports the steady state moments targeted by the model. We set β to 0.9990, to match an annualized real interest rate of 2.40%, along with (annualized) steady state output growth rate of 2%. Innovation step size γ is set to 1.55 to match the creative destruction rate of 3.6%. Howitt (2000) selects this value as it matches the empirical finding that a non-innovating U.S. company loses value at a 3.6-percent annual rate. Capital depreciation rate is set to an annual rate of 10% and steady state price markup is set to 15%. These are commonly used values in the business cycle literature. We calibrate α, δ, and ϱ such that model replicates following (annual) steady state targets: Gross Private Domestic investment to GDP ratio of 17.2%, growth rate of 2%, R&D to GDP ratio of 2%, and Profits to GDP ratio of 6.2%. These are calculated from quarterly NIPA tables over 1947-2007.

Table I.2: Parameters

Steady State Parameters					
	β	λ_p	δ_k	α	γ
	Discount factor	Price s.s. markup	Capital depreciation rate	Capital share	Innovation step size
	0.999	0.15	0.025	0.28	1.55
		ϱ	δ	μ	
Calibrations		Inverse innovation elasticity	Innovation cost parameter	Probability of patent loss	
1. *low* ϱ		1.07	5.88	0.0285	
2. *high* ϱ		3.08	7.47×10^4	0.0	
Parameters Characterizing the Dynamics					
ν	λ_w	θ_p	θ_w	h	$\frac{a''(1)}{a'(1)}$
Inverse Frisch elasticity	Wage s.s. markup	Price Calvo probability	Wage Calvo probability	(Internal) habit	Capital utilization cost
1.00	0.15	0.750	0.750	0.5	4
$\varkappa$	$S"(1)$	ϕ_π	ϕ_y	$1-\frac{1}{\lambda_g}$	
R&D adjustment cost	Investment adjustment cost	Taylor rule inflation response	Taylor rule (normalized) output response	Government spending share	
0.768	0.75	1.50	0.125	0.20	

Notes: The table shows the parameter values of the model for the baseline calibration.

Table I.3: Targets and Model-Implied Values in Calibration of Steady State Parameters

Targets	GDP growth rate	Creative Destruction rate	Real rate	Investment/GDP Ratio	R&D/GDP Ratio	Profits/GI Ratio
Data	2	3.6	2.40	17.18	2	6.50
Model	2	3.6	2.40	17.18	2	6.59

Notes: The table shows the empirical targets and the model-implied values in the calibration of the six steady state parameters. The sample used to compute the data counterparts of the targets is 1948Q1-2007Q4.

We consider two variants of the model to vary the innovation sensitivity. Under first calibration, following

Benigno and Fornaro (2018), we introduce an exogenous probability of patent loss $\mu = 11.4\%$. This implies that value of owning an intermediate goods' patent is modified to:

$$V_t = \Gamma_t + (1 - z_{it} - \mu)\mathbb{E}_t Q_{t,t+1} V_{t+1}$$

μ is chosen in order to match the (annual) R&D depreciation rate of 15%. An exogenous probability of patent loss reduces profitability from successful innovation, and in turn reduces R&D investment. Ceteris paribus, a higher exogenous patent loss probability requires higher returns from R&D investment, and thus lower ϱ. As a result, we find $\varrho = 1.07$. Schumpeterian growth literature following Aghion and Howitt (1992) has largely focused on the analytically tractable case of $\varrho = 1$ (cf. Nuño 2011). There is an extensive empirical literature that estimates this parameter (surveyed in Hall et al. 2010) and finds a relatively wide range $\varrho \in (1.10, 5)$. Low ϱ implies higher sensitivity of innovation probability to R&D investment, which invariably allows the model to generate large growth rate fluctuations.[†] Additionally, we recalibrate the model without the exogenous patent loss to get a calibration with higher $\varrho = 3.08$.

Parameters characterizing Endogenous Propagation

Remaining set of parameters are chosen from the standard business cycle literature, and we closely follow Del Negro et al. (2017) in calibrating these parameters. Inverse Frisch elasticity of labor supply is set to 1, wage markup is set to steady state markup of 15% to mirror the degree of monopolistic competition assumed in the product market ($\lambda_w = 0.15$). Nominal rigidities parameters are chosen, following the empirical evidence of Nakamura and Steinsson (2008) who find an average duration of price and wage contracts to be 4 quarters ($\theta_p = \theta_w = 0.75$). We calibrate habits parameter at $h = 0.5$. Varying these parameters to ranges considered in the literature does not significantly change our results. Investment adjustment cost parameter $S"(1)$ is set to 0.75, consistent with the estimates of price elasticity of investment (in the range of $1.22 - 1.36$) in Eberly (1997) as well as Christiano and Fisher (1998).

As discussed above, we introduce curvature in R&D investment in order to replicate the curvature in the estimated impulse responses. Brown et al. (2009) estimate an Euler equation model for R&D investment at the firm level using Compustat data and find a baseline estimate for $\frac{\varkappa}{2} = 0.384$. Consequently, we set $\varkappa = 0.768$.[†]

[†]The marginal probability of success is decreasing in ϱ, keeping fixed the profitability upon successful innovation. Exogenous patent loss reduces the profitability of successful innovation, for a given probability of success.

[†]They estimate the following equation for firm j, investing R&D $rd_{j,t}$ at time t:

$$rd_{j,t} = \beta_1 rd_{j,t-1} + \beta_2 rd_{j,t-1}^2 + \text{controls } + \text{fixed effects} + \text{error}_{j,t}$$

We interpret β_2 to be our model equivalent of $\frac{\varkappa}{2}$.

Policy Rule parameters and Exogenous shocks

We set the feedback coefficient on inflation and (normalized) output at 1.50 and 0.125 respectively (Taylor, 1993). Steady state government spending share $(1 - \frac{1}{\lambda_g})$ is set to 0.20. We discuss the persistence of shocks in the next exercises.

Appendix I.3.3. Quantitative Assessment

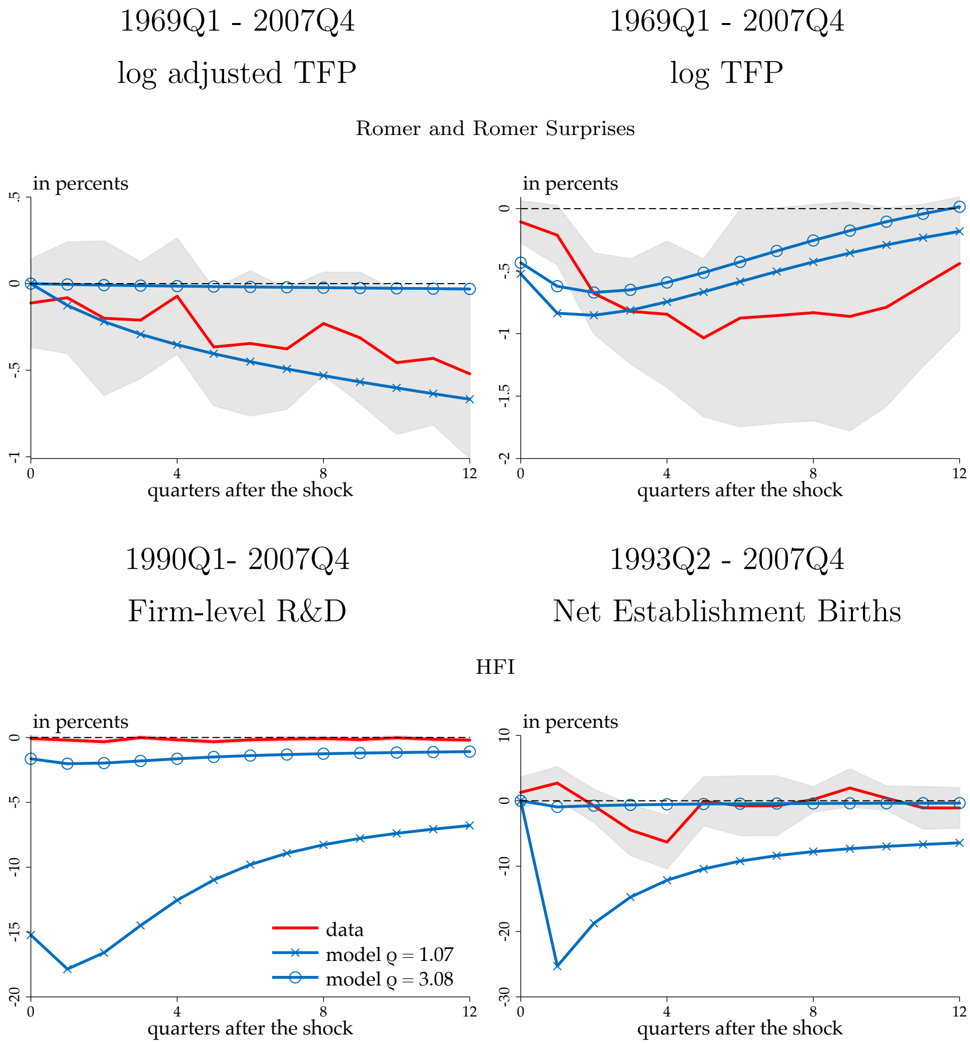
Impulse Response Functions

We shock the economy with a monetary policy shock that generates a 100 basis point (annualized) increase in nominal interest rate on impact. This is the same shock we used in the estimation in Appendix I.1, so that the results are comparable. We choose the persistence of monetary policy shock equal to 0.9, a commonly used estimate in the literature. We report the model IRFs in percent deviations from steady state at time 0. Figure I.10 plots the IRFs for two calibrations of the model against the estimated IRFs for R&D investment, average firm entry, and utilization-adjusted as well as raw TFP. [†] While we do not explicitly model firm entry, we interpret probability of innovation z_t as the average firm entry in the following period consistent with the *creative destruction* aspect of our framework. The monetary policy shocks induce a negative transitory response for R&D investment, average firm entry and a permanent effect on TFP. Because of the presence of adjustment costs in R&D investment, R&D impulse response exhibits an U-shaped response, as seen in the estimated IRFs. R&D investment and firm entry are important sources of TFP growth in the model. While firm entry and R&D investment decline immediately, endogenous slow TFP growth results in a permanently lower level of TFP. Because of absence of technology adoption, TFP monotonically declines to a permanently lower level. As in the data, initial decline in raw TFP exceeds that of the adjusted TFP because of variability in factor utilizations. Overall, the model replicates the estimated dynamic impacts.

Importantly, the impulse response comparisons highlight a tradeoff in calibrating a value for ϱ. Lower ϱ implies higher sensitivity of R&D investment and hence a significant innovation gap emerges. The model, however, is unable to match the empirical response of R&D. Even for the extreme value of $\varrho = 3.08$, the model predicts a larger fall in R&D investment relative to that observed in the data. On the other hand, the model with low ϱ closely replicates the empirical impulse responses for TFP. Given the low responsiveness of R&D investment in the data, the model tends to fit the data under a firm entry interpretation. To the extent firm entry and other forms of investment are significant drivers of TFP growth, there is little reason to treat R&D expenditure in the model solely as the R&D expenditure incurred by publicly-traded firms. In our model, cyclical sensitivities of R&D expenditure and firm-entry are regulated by the same parameter *varrho*. A more micro-founded model that disconnects these important drivers of TFP growth can help rationalize the estimated TFP sensitivity to the estimated R&D and firm-entry responses respectively. Our

[†] In the model, we define raw TFP as sum of two terms (1) deviations in capital utilization from steady state, and (2) deviations in log TFP (pure) from its deterministic trend at time 0.

Figure I.10: Response of Firm Entry, Aggregate R&D and Firm-level R&D to 100 bps increase in Federal Funds Rate



Notes: The figure compares model-implied IRFs to the estimated impulse response functions for utiltization adjusted TFP, raw TFP, firm-level R&D and net establishment births. Time is in quarters. Sample length, and instrument used are denoted at the top of each row. IRFs are computed using a local-projections IV approach. Current and two past-lagged values of log real GDP and inflation rate are used as conditioning variables. Regressions also include past values of the proxy, the federal funds rate, and the dependent variable. Kleibergen-Paap F statistic for weak instruments are reported in the figures. The standard errors are calculated using HAR-Newey-West standard errors. The shaded areas denote 95% confidence intervals. Firm -level R&D regressions also include two lags of assets, short debt, cash, employment, and firm-fixed effects. The standard errors are robust clustered at the firm-level. The model impulse responses are extracted from two calibrations with $\varrho = 1.07$ and $\varrho = 3.08$. In the model, IRFs are traced following a one-time exogenous shock in the federal funds rate of 100 bps (annualized).

short exercise nevertheless is useful in that the bounds on ϱ are likely to be in the range of one and three in reduced form-modeling of endogenous TFP growth.[†]

[†]The range of values for ϱ considered is consistent with wide range of estimates found in aggregate and firm level studies (see Hall et al. 2010).One of the commonly cited estimates come from Griliches (1990), who surveys the literature estimating relationship between R&D and patents (as an indicator of innovation output). Results differ on the estimation strategy: cross-sectional estimates of ϱ lie in range of 1 - 1.67, while within-firm time-series estimates are in the range of 1.5-3.3. Kortum (1993) reports estimates in the

Simulating the Great Recession

We now simulate the model with a liquidity demand shock to study its ability to explain the Great Recession episode. In the model, the liquidity demand shock is characterized by the rise in premium for holding Treasuries - referred to as the *convenience yield* (Krishnamurthy and Vissing-Jorgensen, 2012).The size of the liquidity demand shock is calibrated to generate a rise in the liquidity premium of 180 basis points. This is the preferred parameter choice of Del Negro et al. (2017), who estimate the convenience yield using financial market data.[†] We chose the persistence of the shock to equal 0.938 and 0.95 in two calibrations of ϱ. These are chosen in order to generate a ZLB episode with expected duration of six quarters. This expected duration lies within the range of estimates found in financial market surveys during 2009-2010.

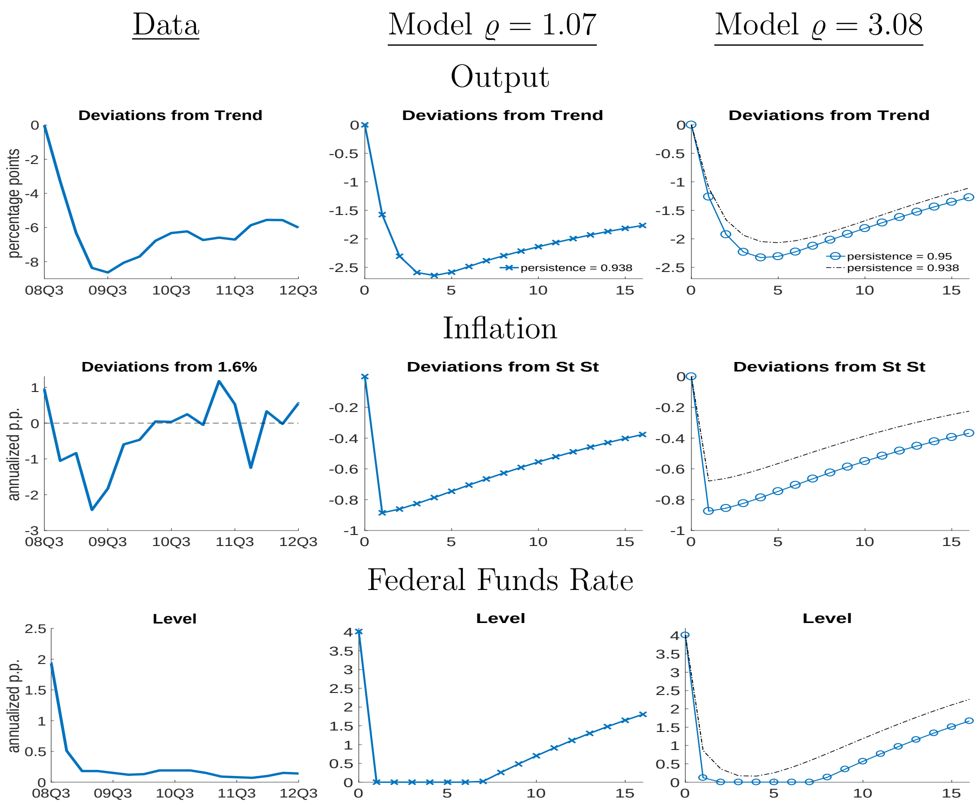
Figure I.11 plots the evolution of output, inflation and nominal interest rate to the calibrated liquidity demand shock and compares it with the data, for sixteen quarters starting in 2008Q3. Column 1 shows the changes in the data relative to 2008Q3 (Lehman Brother's bankruptcy). We report percentage change in output from a linear trend estimated from 2000Q1 to 2007Q4, normalized to zero in 2008Q3. Output is constructed as the log sum of consumption, and investment from the NIPA tables. For inflation, we report the deviation of the annualized percentage change in the GDP deflator from 1.6% annual inflation rate. We chose this number to get the model to match annualized nominal interest rate of 4%. The nominal interest rate is the effective federal funds rate.

Given a relatively modest shock, the model can explain a significant component of the decline in output (-2.6% in the model versus -8.6% in the data). Furthermore, it implies a reduction in inflation of 0.9 percentage points following the shock, compared to an initial drop of 1% in the data. The nominal interest rate hits the zero lower bound, stays at zero for six quarters and sluggishly recovers back. We emphasize the close fit in the dynamics of the model with the data. The model implies no recovery to the 2000Q1-2007Q4 trend, as has been observed in the data. Calibrations of $\varrho = 1.07$ and 3.08 imply a 1.25% and 0.08% permanently lower output respectively, relative to pre-recession trend. In figure I.11, we compare the evolution of consumption, investment and R&D investment with the data. The model replicates the broad empirical pattern of generating more decline in investment relative to consumption. Moreover, it generates a persistent decline in consumption relative to investment. The model with low ϱ (line with crosses) implies a more sluggish recovery in consumption relative to high ϱ (line with circles). Because of higher sensitivity of R&D investment, low ϱ generates a counterfactually large response of R&D investment. In the data, R&D investment declined by 6%, while low ϱ implies a decline of 16%. In contrast, the model with high ϱ

range of (1.3,10).

[†]The results are qualitatively similar, but larger in magnitude, when we calibrated the shock to match rise in spread between AAA and 20 year Treasuries, or the spread between most recently used and older 10 year Treasury bonds of same maturity, called the *on-the-run/ off-the-run* spread.

Figure I.11: Response of Output, Inflation, and the Nominal Interest Rate to the Liquidity Shock



Notes: The figure compares the evolution of output, inflation, and the nominal interest rate in the data (left column) and in the two variants of the model in response to the calibrated liquidity shock (right columns). The data start in 2008Q3. Both data and model are plotted for 16 quarters. Output in the data (top-left) is the sum of consumption and investment, in percentage log-deviations from a linear trend estimated from 2000Q1 to 2007Q4, and is normalized to zero in 2008Q3. Inflation in the data (middle-left) is the annualized quarterly inflation rate of the GDP deflator minus 1.6%. Value of 1.6% is chosen for the model to hit a steady state nominal interest rate of 4%. The interest rate in the data (bottom-left) is the annualized effective Federal Funds Rate. Output in the model (top-right) is the log-deviation from steady state in percentage points. Inflation in the model (middle-right) is expressed in annualized percentage points. The interest rate in the model (bottom-right) is the annualized level of the nominal interest rate in percentage points (the horizontal line is its steady state value).

generates a 1.8% decline in R&D investment.[†]

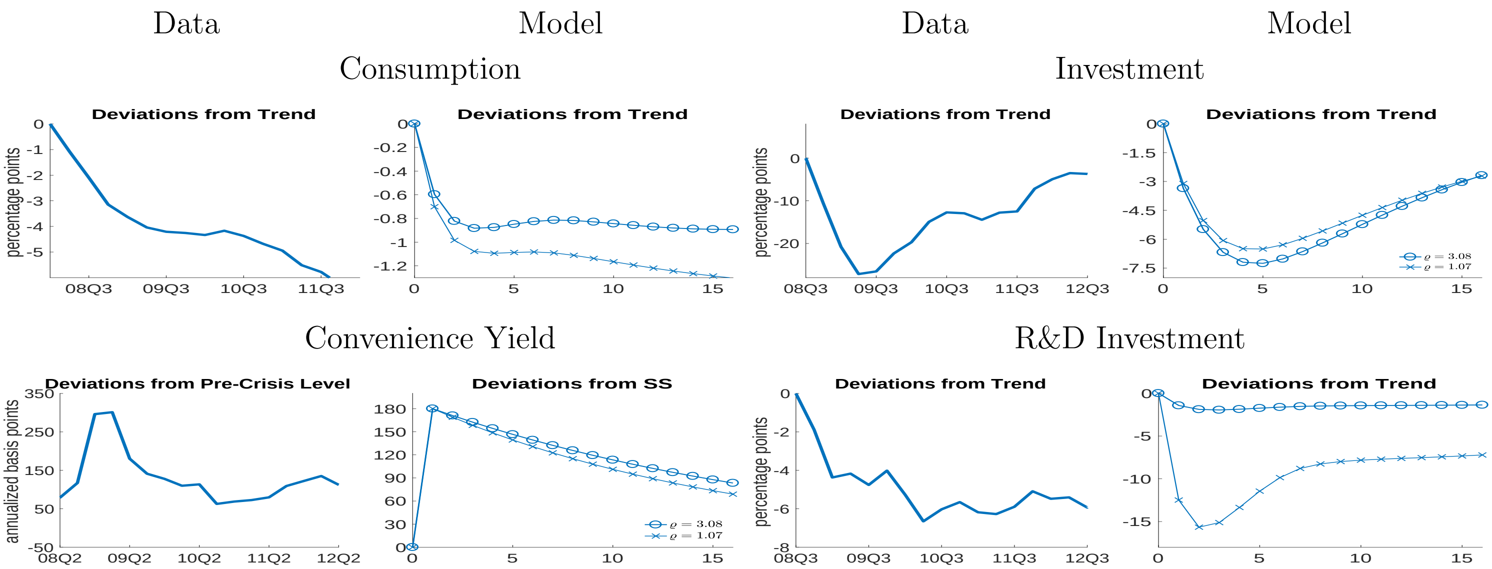
Hysteresis targeting during the Great Recession

How does a hysteresis targeting rule perform in a quantitative model? We assume that the central bank sets interest rate using the following hysteresis-augmented interest rate rule, with $\phi_h = 0.5$:

$$\hat{i}_t = \max\left(-\frac{\bar{i}}{1+\bar{i}}, \phi_\pi \hat{\pi}_t^w + \phi_y(\hat{L}_t - \hat{L}_t^f) + \phi_h h_{t+1} + \hat{\varepsilon}_t^i\right) \tag{I.2}$$

[†]Note that persistence of the simulated shock is calibrated such that the expected duration of ZLB is six quarters. Consequently, the recession is less severe. In the Appendix Appendix J.10, we show that a more persistent shock where the ZLB is expected to bind for twelve quarters performs better at replicating the drop in output, inflation, consumption and investment in the data. Moreover, the drop in consumption is more persistent and less severe than output and investment. Because of pro-cyclicality of R&D investment, a more severe recession, however, also implies a larger drop in R&D.

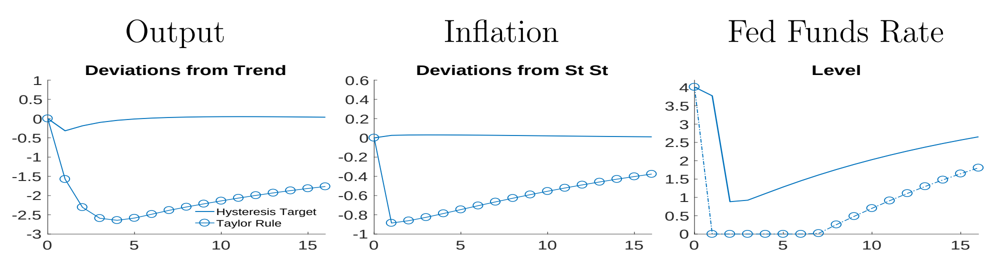
Figure I.12: Response of Consumption, Investment, R&D Investment, and Convenience Yield to the Liquidity Shock



Note: The figure compares the evolution of consumption, investment, R&D investment, and convenience yield in the data (left column) and in the model in response to the calibrated liquidity shock (right column). The data start in 2008Q3. Both data and model are plotted for 16 quarters. Consumption in the data (top-left) is total consumption minus durable consumption. Investment in the data (top-middle-left) is investment plus durable consumption minus Intellectual Property Investment. R&D Investment in the data (bottom-middle-left) is the Intellectual Property Investment. These three variables are expressed in percentage log-deviations from a linear trend estimated from 2000Q1 to 2007Q4, and are normalized to zero in 2008Q3. The convenience yield in the data (bottom-left) is in annualized basis points (produced by (Del Negro et al., 2017)). Consumption (top-right), investment (top-middle-right), and R&D investment (bottom-middle-right) in the model are log-deviations from steady state in percentage points. The convenience yield in the model (bottom-right) is the annualized absolute deviation from steady state expressed in basis points.

where superscript f denotes the flexible-price-wage allocation, hysteresis $h_{t+1} = h_t + \hat{g}^f_{t+1}$, where g_{t+1} is determined by R&D investments in period t.

Figure I.13: Hysteresis Targeting: Response of Output, Inflation, and the Nominal Interest Rate to the Liquidity Shock



Notes: The figure compares the evolution of output, inflation, and the nominal interest rate under Hysteresis targeting rule and assumed Taylor rule in the model with $\varrho = 1.07$ in response to the calibrated liquidity shock. All graphs are plotted for 16 quarters. Output in the model (top-right) is the log-deviation from steady state in percentage points. Inflation in the model (middle-right) is expressed in annualized percentage points, deviation from steady state value of 1.6%. The interest rate in the model (bottom-right) is the annualized level of the nominal interest rate in percentage points (the horizontal line is its steady state value). Hysteresis targeting rule is implemented by adding an additional term called the *hysteresis* with a coefficient of 0.5. Hysteresis is defined as sum of all endogenous growth rate deviations induced by history of shocks at time t.

Figure I.13 compares the evolution of output, inflation and interest rate under the above interest rate rule with $\phi_h = 0.5$ (Hysteresis targeting) to rule with $\phi_h = 0$ (Standard Taylor rule). We only plot the figures for the case of $\varrho = 1.07$. The results are similar in the case of $\varrho = 3.08$, although the permanent output shortfall is significantly smaller in that setting. Output falls by only 0.3% under hysteresis targeting compared to the 2.6% drop under Taylor rule. Inflation and federal funds rate are positive (in contrast to Taylor rule). An explicit commitment to targeting permanent output shortfalls creates inflationary expectations, which lowers the natural interest rate. Higher expected inflation provides more room for the central bank to offset declines in natural interest rates, as the central bank in this exercise has the power to reduce the impact of the shock by lowering the nominal interest rate. This example illustrates that the *hysteresis bias* embedded in a standard Taylor rule has quantitatively significant implications for the permanent level of output.

Appendix J. Derivation and details for the medium scale DSGE model

We follow Howitt and Aghion (1998) and Aghion and Howitt (2008) in introducing capital in the endogenous growth framework. We however extend our model to allow for investment adjustment costs in sync with the New Keynesian literature following (Christiano et al., 2005), Smets and Wouters (2007) and Justiniano et al. (2013). The new ingredients are (i) a monopolistically competitive retail sector that sets prices in a staggered fashion, (ii) endogenous capital accumulation by households subject to investment adjustment costs, (iii) habit formation in consumption, (iv) variable capital utilization rate, and (v) partial indexation of prices and wages to the respective lagged inflation rates. We discuss these in turn:

Appendix J.1. Monopolistically Competitive Retailers

There is a continuum of monopolistically competitive retailers that sell the final good $Y_t(k)$. These goods can be aggregated into a Dixit-Stiglitz composite Y_t as follows:

$$Y_t = \left[\int_0^1 Y_t(k)^{\frac{1}{1+\lambda_{p,t}}}\, dk\right]^{1+\lambda_{p,t}}$$

where $\lambda_{p,t} > 0$ is the (time-varying) price markup. We assume that $\lambda_{p,t}$ follows the exogenous ARMA process:

$$\log \lambda_{p,t} = (1-\rho_p)\log \lambda_p + \rho_p \log \lambda_{p,t-1} + \epsilon_t^p - \mu_p \epsilon_{t-1}^p; \quad \epsilon_t^p \sim N(0, \sigma_p)$$

Each retailer k purchases one unit of intermediate good composite $Y_t(k, m)$ at a given price of P_t^M to package it into one unit of final good and is assumed to set prices on a staggered basis following Calvo (1983). With probability $(1-\theta_p)$, a retailer gets to reset its price. It solves the following problem:

$$\max_{P_t(k)} \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\theta_p)^s Q_{t,t+s}\left[P_t(k)\Pi_{t,t+s} - P_{t+s}^M\right] Y_{t+s}(k)$$

subject to demand for its product

$$Y_{t+s}(k) = \left(\frac{P_t(k)\Pi_{t,t+s}}{P_{t+s}}\right)^{-\frac{1+\lambda_{p,t+s}}{\lambda_{p,t+s}}}$$

where the stochastic discount factor period $t+s$ is given by:

$$Q_{t,t+s} = \beta \frac{\Lambda_{t+s}}{\Lambda_t}\frac{P_t}{P_{t+s}}$$

where Λ_t is the marginal utility of consumption defined later and

$$\Pi_{t,t+s} \equiv \prod_{b=1}^{s} \left(\pi_{ss}^{1-\iota_p} \pi_{t+b-1}^{\iota_p}\right)$$

is the automatic adjustment that firms make to their price when they do not get to reset them, $\iota_p \in (0,1)$ is the indexation coefficient and π_{ss} is the steady state price inflation rate. Let $\tilde{P}_t$ be the reset price at time t. The first order condition is :

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta\theta_p)^s Q_{t,t+s} \left[\tilde{P}_t \Pi_{t,t+s} - (1+\lambda_{p,t+s}) P_{t+s}^M\right] Y_{t+s}(k) = 0$$

The law of motion of the aggregate price index P_t is given by:

$$P_t^{\frac{1}{\lambda_{p,t}}} = (1-\theta_p)(\tilde{P}_t)^{\frac{1}{\lambda_{p,t}}} + \theta_p \left(\pi_{t-1}^{\iota_p} \pi_{ss}^{1-\iota_p} P_{t-1}\right)^{\frac{1}{\lambda_{p,t}}}$$

Appendix J.2. Perfectly Competitive Composite Good Production

Each of the intermediate good composites is produced by a perfectly competitive firm that uses a CES composite of labor and secondary intermediate goods.[†] As a result, all intermediate good firms are identical and we omit the subscripts (k, m) and simply denote the intermediate output at Y_t^m.

$$Y_t^m = M_t L_t^{1-\alpha} \int_0^1 A_{it} x_{it}^{\alpha} di,$$

where each x_{it} is the flow of intermediate product i used at time t, and the productivity parameter A_{it} reflects the quality of that product and M_t is the aggregate (stationary) productivity shock which follows the process:

$$\log M_t = (1-\rho_m)\log M_t + \rho_m \log M_{t-1} + \epsilon_t^m; \quad \epsilon_t^m \sim N(0, \sigma_m)$$

The composite good producer's maximization problem is is as follows

$$\max_{L_t, \{x_{it}\}_{i\in[0,1]}} \left\{ P_t^m M_t L_t^{1-\alpha} \int_0^1 A_{it} x_{it}^{\alpha} di - W_t L_t - \int_0^1 p_{it} x_{it} di \right\}$$

Solving this gives the (inverse) factor demands:

$$p_{it} = \alpha P_t^m M_t L_t^{1-\alpha} A_{it} x_{it}^{\alpha-1} \tag{J.1}$$

$$W_t = (1-\alpha) P_t^m \frac{Y_t^m}{L_t} \tag{J.2}$$

Appendix J.3. Monopolist Intermediate Good Producer

Intermediate good producers are monopolists and use capital to produce one unit of intermediate good. Following Howitt and Aghion (1998), we assume the following production function for the intermediate good:

$$x_{it} = \frac{K_{it}}{A_{it}}$$

[†]Such a convoluted market structure is assumed to introduce price -rigidity in a staggered fashion. Basically, there is a single consumption good that is produced by a perfectly competitive firm, but is retailed by monopolistically competitive retailers in different packaging.

The intermediate monopolistic firm sets prices flexibly every period in order to maximize profits:.

$$\max_{p_{it}} \quad (1-\tau_t^p)p_{it}x_{it} - R_t^K K_{it}$$

subject to the demand for the intermediate good (eq J.1). τ_t^p is the sales tax/subsidy imposed on the monopolist's price. Further, we assume that there is a competitive fringe in every sector that faces a marginal cost of $\gamma^{1-\alpha}A_{it}R_t^K$, where γ is the step-size of innovation, discussed in following subsection. As a result, the intermediate monopolist cannot charge a price higher than $p_{it} = \chi A_{it}R_t^K$. In equilibrium, the monopolist charges a price given by:

$$p_{it} = \zeta A_{it}R_t^K \equiv \min\left(\gamma^{1-\alpha}, \frac{1}{(1-\tau^p)\alpha}\right) A_{it}R_t^K$$

This yields

$$x_{it} = \frac{K_t}{A_t} = \left(\frac{\frac{\alpha}{\zeta}P_t^m M_t L_t^{1-\alpha}}{R_t^K}\right)^{\frac{1}{1-\alpha}}, \quad R_t^K = \frac{\alpha}{\zeta}\frac{P_t^m Y_t^m}{K_t}$$

and profits are given by $\Gamma_t(A_{it}) = (\zeta-1)\frac{\alpha}{\zeta}\frac{P_t^m Y_t^m A_{it}}{A_t}$. Define aggregate productivity $A_t \equiv \int_0^1 A_{it}di$. Substituting for the intermediate goods' production levels, we can rewrite the production function purely in the form of aggregates:

$$Y_t^m = M_t(A_tL_t)^{1-\alpha}K_t^\alpha \tag{J.3}$$

Define $k_t = \frac{K_t}{A_t}$ and $y_t^m = \frac{Y_t^m}{A_t} = M_t k_t^\alpha l_t^{1-\alpha}$.

Appendix J.4. Innovation and research arbitrage

There is a single entrepreneur in each sector who spends final output in research . The entrepreneur at time t decides her research inputs and if successful, she gets to be the intermediate monopolist in the following period producing goods with productivity $A_{it+1} = \gamma A_{it}$. She is successful with probability $\Omega_t z_{it}$, where Ω_t is the exogenous shock to innovation success and and is assumed to follow the following process:

$$\log\Omega_t = \rho_\Omega \log\Omega_{t-1} + \epsilon_t^\Omega; \quad \epsilon_t^\Omega \sim N(0,\sigma_\Omega)$$

z_{it} is the innovation intensity chosen by the entrepreneur. In order to achieve this, she needs to spend the amount of final good[†]

$$R_{it} = c(z_{it})A_{it} + S^r\left(\frac{R_t}{(1+g_{ss})R_{t-1}}\right)A_t$$

in research, where $c(z_t) \equiv \delta z_t^\varrho; \varrho > 1$. $S^r(\cdot)$ denote adjustment costs in R&D that the entrepreneur takes as given. We assume $S^r(1) = 0$ and $\frac{\partial S^r}{\partial R}(1) > 0$. These adjustment costs generate a hump shape response for R&D expenditure. These costs are similar to those considered by Aghion et al. (2010) since these enter additively and do not affect the first-order condition for entrepreneur. Entrepreneur maximizes the net expected profits from investing in research :

$$\max_{z_{it}\in[0,1]}\{\Omega_t z_{it} Q_{t,t+1} V_{t+1}(\gamma A_{it}) - (1-\tau_t^r)P_t R_{it}\}$$

where the lifetime discounted profits are given by the value function:

$$V_t(A_{it}) = \Gamma_t(A_{it}) + (1-\Omega_t z_{it})\mathbb{E}_t Q_{t,t+1}V_{t+1}(A_{it})$$

[†]This could further be generalized to allow for adoption probability for this entrepreneur's technology in the next period, which would better match the data. Secondly, we can also add a financial frictions constraint to get more action.

Because of the linearity of production function, as we showed above in the Appendix A, the Value function is also linear in productivity. Writing the normalized Value function as $\tilde{V}_{it} \equiv \frac{V_{it}}{P_t A_{it}}$ and focusing on the symmetric equilibrium, we solve for interior solution (where $z_t > 0$):

$$\varrho z_t^{\varrho-1} = \beta \frac{\Lambda_{t+1}}{\Lambda_t} \frac{\gamma \Omega_t \tilde{V}_{t+1}}{(1-\tau_t^r)\delta} \tag{J.4}$$

Total amount of the final good used in research and innovation:

$$R_t = \int_0^1 R_{it} di = \left(c(z_t) + S^r \left(\frac{R_t}{(1+g_{ss})R_{t-1}} \right) \right) A_t$$

Appendix J.5. Households & Wage Setting

Appendix J.5.1. Households

Each household supplies differentiated labor indexed by j. Household j chooses consumption C_t, risk-free nominal bonds B_t, investment I_t and capital utilization u_t to maximize the utility function:

$$\mathbb{E}_t \Sigma_{s=0}^{\infty} \beta^j \left[\log(C_{t+s} - hC_{t+s-1}) - \frac{\omega}{1+\nu} L_{t+s}(j)^{1+\nu} + \xi_t \frac{B_{t+1}}{P_t} \right]$$

where h is the degree of habit formation, $\nu > 0$ is the inverse Frisch elasticity of labor supply, $\omega > 0$ is a parameter that pins down the steady-state level of hours, the discount factor β satisfies $0 < \beta < 1$ and ξ_t is the liquidity demand shock. We assume that in the steady state $\xi = 0$ .We assume perfect consumption risk sharing across the households. As a result, household's budget constraint in period t is given by

$$P_t C_t + P_t I_t + B_{t+1} = B_t(1+i_t) + B_t^S(j) + (1+\tau^w) W_t L_t(j) + \Gamma_t + T_t + R_t^K u_t K_t^u - P_t a(u_t) K_t^u \tag{J.5}$$

where I_t is investment, $B_t^S(j)$ is the net cash-flow from household j's portfolio of state-contingent securities. Labor income $W_t L_t(j)$ is subsidized at a fixed rate τ_w. Households own an equal share of all firms, and thus receive Γ_t dividends from profits. Finally, each household receives a lump-sum government transfer T_t. Since households own the capital and choose the utilization rate, the amount of effective capital that the households rent to the firms at nominal rate R_t^K is :

$$K_t = u_t K_t^u$$

The (nominal) cost of capital utilization is $P_t a(u_t)$ per unit of physical capital. As in the literature (SW 2007, JPT 2010) we assume $a(1) = 0$ in the steady state and $a'' > 0$. Following CEE 2005, we assume investment adjustment costs in the production of capital. Law of motion for capital is as follows:

$$K_{t+1}^u = \upsilon_t \left[1 - S\left(\frac{I_t}{(1+g_{ss})I_{t-1}} \right) \right] I_t + (1-\delta_k) K_t^u$$

where g_{ss} is the steady state growth rate of productivity, ϵ_t^i is a shock to the relative price of investment and In the steady state $S(1) = S'(1) = 0$, $S'' > 0$. JPT consider this as shock to *marginal efficiency of investment* (MEI) and is assumed to follow the following process:

$$\log \upsilon_t = \rho_\upsilon \log \upsilon_{t-1} + \epsilon_t^\upsilon; \quad \epsilon_t^\upsilon \sim N(0, \sigma_\upsilon)$$

Utility maximization delivers the first order condition linking the inter-temporal consumption smoothing to the marginal utility of holding the risk-free bond

$$1 = \beta \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} (1+i_t) \frac{P_t}{P_{t+1}} \right] + \Lambda_t^{-1} \xi_t \tag{J.6}$$

The stochastic discount factor in period $t+1$ is given by:

$$Q_{t,t+1} = \beta \frac{\Lambda_{t+1}}{\Lambda_t} \frac{P_t}{P_{t+1}}$$

where Λ_t is the marginal utility of consumption given by:

$$\Lambda_t = \frac{1}{C_t - hC_{t-1}} - \frac{h\beta}{C_{t+1} - hC_t}$$

The household does not choose hours directly. Rather each type of worker is represented by a wage union who sets wages on a staggered basis. Consequently the household supplies labor at the posted wages as demanded by firms.

We introduce capital accumulation through households. Solving household problem for investment and capital yields the Euler condition for capital:

$$q_t = \beta \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \left(\frac{R^K_{t+1}}{P_{t+1}} u_{t+1} - a(u_{t+1}) + q_{t+1}(1-\delta_k) \right) \right]$$

where the (relative) price of installed capital q_t is given by

$$q_t \upsilon_t \left[1 - S\left(\frac{I_t}{(1+g_{ss})I_{t-1}} \right) - S'\left(\frac{I_t}{(1+g_{ss})I_{t-1}} \right) \frac{I_t}{(1+g_{ss})I_{t-1}} \right] + \beta \frac{\Lambda_{t+1}}{\Lambda_t} q_{t+1} \upsilon_{t+1} \frac{1}{(1+g_{ss})} \left(\frac{I_{t+1}}{I_t} \right)^2 S'\left(\frac{I_{t+1}}{(1+g_{ss})I_t} \right) = 1$$

Choice of capital utilization rate yields:

$$\frac{R^K_t}{P_t} = a'(u_t)$$

Appendix J.5.2. Wage Setting

Wage Setting follows Erceg et al. (2000) and is relatively standard. Perfectly competitive labor agencies combine j type labor services into a homogeneous labor composite L_t according to a Dixit-Stiglitz aggregation:

$$L_t = \left[\int_0^1 L_t(j)^{\frac{1}{1+\lambda_{w,t}}} dj \right]^{1+\lambda_{w,t}}$$

where $\lambda_{w,t} > 0$ is the (time-varying) nominal wage markup. We assume that $\lambda_{w,t}$ follows the exogenous ARMA process:

$$\log \lambda_{w,t} = (1-\rho_w) \log \lambda_w + \rho_w \log \lambda_{w,t-1} + \epsilon^w_t - \mu_w \epsilon^w_{t-1}; \quad \epsilon^w_t \sim N(0, \sigma_w)$$

Labor unions representing workers of type j set wages on a staggered basis following Calvo (1983), taking given the demand for their specific labor input:

$$L_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} L_t, \qquad \text{where } W_t = \left[\int_0^1 W_t(j)^{\frac{-1}{\lambda_{w,t}}} dj \right]^{-\lambda_{w,t}}$$

In particular, with probability $1-\theta$, the type-j union is allowed to re-optimize its wage contract and it chooses $\tilde{W}$ to minimize the disutility of working for laborer of type j, taking into account the probability that it will not get to reset wage in the future. If a union is not allowed to optimize its wage rate, it adjusts wage at steady state wage inflation $\bar{\Pi}^w$ rate. Workers supply whatever labor is demanded at the posted wage. The first order condition for this problem is given by:

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta\theta_w)^s \Lambda_{t+s} \left[(1+\tau^W_t) \tilde{W} \Pi^w_{t,t+s} - (1+\lambda_{w},t) \omega \frac{L^{\nu}_{t+s}(j)}{\Lambda_{t+s}} \right] L_{t+s}(j) = 0 \tag{J.7}$$

where

$$\Pi^w_{t,t+s} \equiv \prod_{b=1}^{s} (\pi^w_{ss})$$

By the law of large numbers, the probability of changing the wage corresponds to the fraction of types who actually change their wage. Consequently, the nominal wage evolves according to:

$$W_t^{\frac{1}{\lambda_{w,t}}} = (1-\theta_w)(\tilde{W}_t)^{\frac{1}{\lambda_{w,t}}} + \theta_w \left(\pi^w_{ss} W_{t-1}\right)^{\frac{1}{\lambda_{w,t}}}$$

where the nominal wage inflation and price inflation are related to each other :

$$\pi^w_t = \frac{W_t}{W_{t-1}} = \frac{w_t}{w_{t-1}} \frac{1}{\pi_t} \frac{1}{1+g_t}$$

where $\pi_t \equiv \frac{P_t}{P_{t-1}}$ is the inflation rate, $w_t \equiv \frac{W_t}{P_t A_t}$ is the productivity adjusted real wage and g_t is the (endogenous) productivity growth rate.

Appendix J.6. TFP and growth rate

Aggregate (endogenous) productivity follows:

$$A_t = \int_0^1 A_{it} di = \int_0^1 [\Omega_{t-1} z_{t-1} \gamma A_{it-1} + (1 - \Omega_{t-1} z_{t-1}) A_{it-1}] di = A_{t-1} + \Omega_{t-1} z_{t-1} (\gamma - 1) A_{t-1}$$

The growth rate of the productivity :

$$g_t = \frac{A_t - A_{t-1}}{A_{t-1}} = \Omega_{t-1} z_{t-1} (\gamma - 1)$$

Measured TFP (total factor productivity) is given by product of stationary exogenous component and the non-stationary endogenous component :

$$TFP_t = M_t \times A_t$$

Appendix J.7. Government

The central bank follows a Taylor rule in setting the nominal interest rate. It responds to deviations in inflation, output and output growth rate from time-t natural allocations.

$$\frac{1+i_t}{1+i_{ss}} = \left(\frac{\pi_t}{\pi_{ss}}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^{f,t}}\right)^{\phi_y} \epsilon_t^{mp}$$

where i_{ss} is the steady state nominal interest rate, $Y_t^{f,t}$ is the time-t natural output, and $\epsilon_t^{mp} \sim N(0, \sigma_r)$ is an AR(1) monetary policy shock with persistence ρ_R.

We assume government balances budget every period:

$$P_t T_t = \tau^p \int_0^1 p_{it} x_{it} di + \tau_t^r P_t R_t + \tau^w W_t L_t + P_t G_t$$

where G_t is the government spending, which is determined exogenously as as a fraction of GDP

$$G_t = \left(1 - \frac{1}{\lambda_t^g}\right) Y_t$$

where the government spending shock follows the process:

$$\log \lambda_t^g = (1-\rho_g)\lambda^g + \rho_g \log \lambda_{t-1}^g + \epsilon_t^g; \quad \epsilon_t^g \sim N(0, \sigma_g)$$

Appendix J.8. Market Clearing

$$Y_t = C_t + I_t + R_t + a(u_t)K_t^u + G_t$$

Appendix J.9. Stationarizing the system

We normalize the following variables:

$$y_t = Y_t/A_t$$
$$y_t^m = Y_t^m/A_t$$
$$c_t = C_t/A_t$$
$$k_t = K_t/A_t$$
$$k_t^u = K_t^u/A_{t-1}$$
$$\mathbb{I}_t = I_t/A_t \quad \text{capital investment}$$
$$\mathbb{R}_t = R_t/A_t \quad \text{R\&D investment}$$
$$\mathbb{G}_t = G_t/A_t \quad \text{Govt Spending}$$
$$w_t = W_t/(A_t P_t)$$
$$p_t^m = P_t^m/P_t$$
$$r_t^k = R_t^k/P_t$$
$$\lambda_t = \Lambda_t A_t$$
$$\xi_t' = \xi_t A_t$$
$$\tilde{\Gamma}_t \equiv \frac{\Gamma_t}{P_t A_t}$$

Further note that because of the linearity assumption in the production of final goods, the Value function is a linear function in productivity with which an entrepreneur enters the sector:

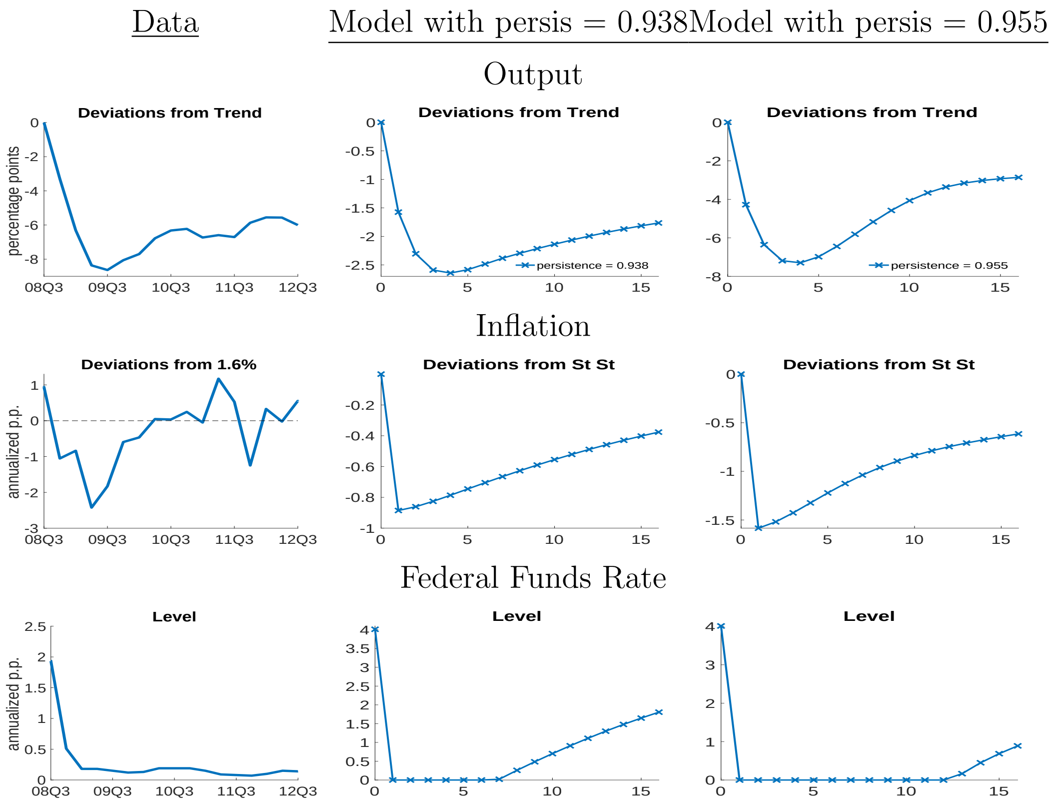
$$\tilde{V}_t \equiv \frac{V_t(A_{it})}{P_t A_{it}} = \tilde{\Gamma}_t + (1 - z_t)\mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \tilde{V}_{t+1}$$

where $\tilde{V}$ is normalized by the productivity with which the entrepreneur enters the sector.

Appendix J.10. Matching the Great Recession: II

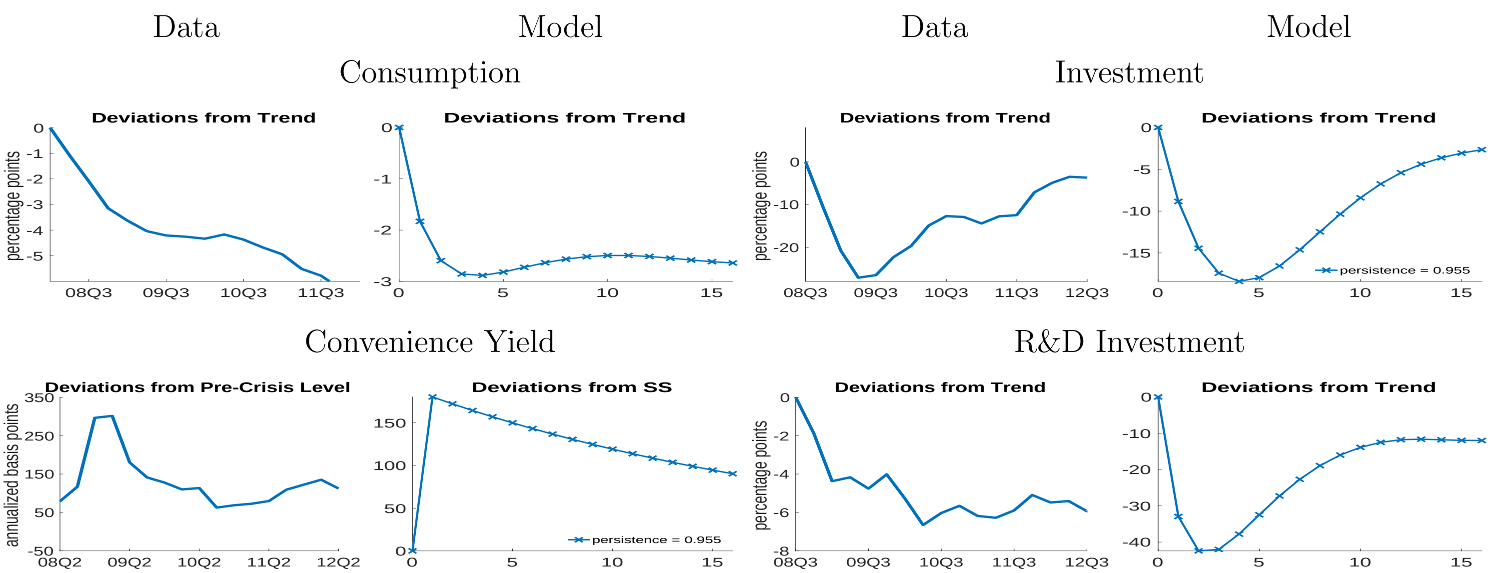
In the main text, we choose a conservative persistence for the liquidity demand shock such that the ZLB is expected to bind for six quarters. Here, we choose a persistence such that the expected duration of the ZLB is 12 quarters, i.e 3 years. We show that the model is better able to match the empirical moments. Figure J.14 plots the evolution of Output, Inflation and Federal funds Rate from 2008Q3 till 2012Q3. The first two columns on the left reproduce the results reported in Figure I.11 for comparison. Column 3 (rightmost) reports results from the model with a more persistent liquidity demand shock. The shock is calibrated such that convenience yield rises by 180 bps on impact. The nominal interest rate hits the ZLB under the Taylor rule and stays there for 12 quarters. Output drops by 7.30%, and Inflation drops by 1.58%.Contrast this with the data where output drops by 8.6% and Inflation drops by 2%. Thus, the liquidity demand shock can explain 84% of the drop in output and 79% of the observed drop in inflation. Figure J.15 plots consumption, and investment under a more persistent shock. In the data, consumption and capital investment drop by 4.34% and 27% respectively. In the model with more persistent liquidity demand shock, the drop in consumption and capital investment are 2.88% and 18.40% respectively. Thus, the liquidity demand shock can explain 66% of the observed drop in consumption and 68% of the observed drop in investment. Importantly, the model produces a faster recovery in investment as observed in the data, while consumption recovers sluggishly. As noted in the main text, the model generates counterfactually high responsiveness of R&D investment. This persistent liquidity demand shock reduces long-run output by 2.50%, through a slowdown in endogenous productivity growth.

Figure J.14: Response of Output, Inflation, and the Nominal Interest Rate to the Liquidity Shock



Notes: The figure compares the evolution of output, inflation, and the nominal interest rate in the data (left column) and in the two variants of the model in response to the calibrated liquidity shock (right columns). The first two columns plot the data and model with exogenous patent loss as in Figure I.11. Column 3 plots these variables in response to a more persistent liquidity demand shock. The data start in 2008Q3. Both data and model are plotted for 16 quarters. Output in the data (top-left) is the sum of consumption and investment, in percentage log-deviations from a linear trend estimated from 2000Q1 to 2007Q4, and is normalized to zero in 2008Q3. Inflation in the data (middle-left) is the annualized quarterly inflation rate of the GDP deflator minus 1.6%. Value of 1.6% is chosen for the model to hit a steady state nominal interest rate of 4%. The interest rate in the data (bottom-left) is the annualized effective Federal Funds Rate. Output in the model (top-right) is the log-deviation from steady state in percentage points. Inflation in the model (middle-right) is expressed in annualized percentage points. The interest rate in the model (bottom-right) is the annualized level of the nominal interest rate in percentage points (the horizontal line is its steady state value).

Figure J.15: Response of Consumption, Investment, R&D Investment, and Convenience Yield to the Liquidity Shock with 12 quarters expected ZLB duration



Note: The figure compares the evolution of consumption, investment, R&D investment, and convenience yield in the data (left column) and in the model in response to the calibrated liquidity shock (right column). Column 1 plots the data counterpart of these variables as reported in Figure I.12. Column 2 plots the model evolution under a more persistent liquidity demand shock. The ZLB binds for 12 quarters. The data start in 2008Q3. Both data and model are plotted for 16 quarters. Consumption in the data (top-left) is total consumption minus durable consumption. Investment in the data (top-middle-left) is investment plus durable consumption minus Intellectual Property Investment. R&D Investment in the data (bottom-middle-left) is the Intellectual Property Investment. These three variables are expressed in percentage log-deviations from a linear trend estimated from 2000Q1 to 2007Q4, and are normalized to zero in 2008Q3. The convenience yield in the data (bottom-left) is in annualized basis points (produced by (Del Negro et al., 2017)). Consumption (top-right), investment (top-middle-right), and R&D investment (bottom-middle-right) in the model are log-deviations from steady state in percentage points. The convenience yield in the model (bottom-right) is the annualized absolute deviation from steady state expressed in basis points.

References

Aghion, P., Angeletos, G.M., Banerjee, A., Manova, K., 2010. Volatility and growth: Credit constraints and the composition of investment. Journal of Monetary Economics 57, 246–265.

Aghion, P., Howitt, P., 1992. A model of growth through creative destruction. Econometrica 60.

Aghion, P., Howitt, P.W., 2008. The Economics of Growth. MIT Press.

Baxter, M., King, R.G., 1999. Measuring business cycles: approximate band-pass filters for economic time series. Review of economics and statistics 81, 575–593.

Benigno, G., Fornaro, L., 2018. Stagnation traps. The Review of Economic Studies 85, 1425–1470.

Brown, J.R., Fazzari, S.M., Petersen, B.C., 2009. Financing innovation and growth: Cash flow, external equity, and the 1990s r&d boom. The Journal of Finance 64, 151–185.

Calvo, G.A., 1983. Staggered prices in a utility-maximizing framework. JME .

Chetty, R., Guren, A., Manoli, D., Weber, A., 2016. Does indivisible labor explain the difference between micro and macro elasticities? a meta-analysis of extensive margin elasticities, University of Chicago Press.

Christiano, L., Eichenbaum, M., Evans, C., 2005. Nominal rigidities and the dynamic effects of a shock to monetary policy. Journal of Political Economy 113.

Christiano, L., Fisher, J.D., 1998. Stock market and investment good prices: Implications for macroeconomics. Technical Report.

Christiano, L.J., Fitzgerald, T.J., 2003. The band pass filter. international economic review 44, 435–465.

Chung, H., Herbst, E., Kiley, M.T., 2015. Effective monetary policy strategies in new keynesian models: A reexamination. NBER Macroeconomics Annual 29, 289–344.

Correia, I., Farhi, E., Nicolini, J.P., Teles, P., 2013. Unconventional fiscal policy at the zero bound. American Economic Review 103, 1172–1211.

Cúrdia, V., Ferrero, A., Ng, G.C., Tambalotti, A., 2015. Has us monetary policy tracked the efficient interest rate? Journal of Monetary Economics 70, 72–83.

Decker, R., Haltiwanger, J., Jarmin, R.S., Miranda, J., 2014. The secular decline in business dynamism in the us .

Del Negro, M., Eggertsson, G., Ferrero, A., Kiyotaki, N., 2017. The great escape? a quantitative evaluation of the fed's non-standard policies. The American Economic Review .

Denes, M., Eggertsson, G.B., Gilbukh, S., 2013. Deficits, public debt dynamics and tax and spending multipliers. The Economic Journal 123, F133–F163.

Eberly, J.C., 1997. International evidence on investment and fundamentals. European Economic Review 41, 1055–1078.

Edge, R., 2003. A utility-based welfare criterion in a model with endogenous capital accumulation. Technical Report. Board of Governors of the Federal Reserve System (US).

Eggertsson, G.B., 2011. What fiscal policy is effective at zero interest rates?, in: NBER Macroeconomics Annual 2010, Volume 25. University of Chicago Press, pp. 59–112.

Eggertsson, G.B., Garga, V., 2019. Sticky prices versus sticky information: Does it matter for policy paradoxes? Review of Economic Dynamics 31, 363–392.

Eggertsson, G.B., Mehrotra, N.R., Singh, S.R., Summers, L.H., 2016. A Contagious Malady? Open Economy Dimensions of Secular Stagnation. Technical Report. National Bureau of Economic Research.

Eggertsson, G.B., Woodford, M., 2003. Zero bound on interest rates and optimal monetary policy. Brookings Papers on Economic Activity 2003, 139–233.

Erceg, C.J., Henderson, D.W., Levin, A.T., 2000. Optimal monetary policy with staggered wage and price contracts. Journal of Monetary Economics 46, 281–313.

Fernald, J.G., 2014. A quarterly, utilization-adjusted series on total factor productivity. Federal Reserve Bank of San Francisco Working paper 19, 2014.

Galí, J., 2015. Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications. Princeton University Press.

Gertler, M., Karadi, P., 2015. Monetary policy surprises, credit costs, and economic activity. American Economic Journal: Macroeconomics 7, 44–76.

Gorodnichenko, Y., Weber, M., 2016. Are sticky prices costly? evidence from the stock market. American Economic Review 106, 165–199.

Griliches, Z., 1990. Patent statistics as economic indicators: A survey. Journal of Economic Literature 28, 1661–1707.

Gürkaynak, R.S., Sack, B., Swanson, E., 2005. The sensitivity of long-term interest rates to economic news: evidence and implications for macroeconomic models. The American Economic Review 95, 425–436.

Hall, B., Mairesse, J., Mohnen, P., 2010. Measuring the returns to r&d, in: Handbook of Economics of Innovation, Volume 2. Elsevier. chapter Chapter 24, pp. 1033–1082.

Howitt, P., 2000. Endogenous growth and cross-country income differences. American Economic Review , 829–846.

Howitt, P., Aghion, P., 1998. Capital accumulation and innovation as complementary factors in long-run growth. Journal of Economic Growth 3, 111–130.

Jordà, Ò., Schularick, M., Taylor, A.M., 2019. The effects of quasi-random monetary experiments. Journal of Monetary Economics .

Jordà, Ò., Singh, S.R., Taylor, A.M., 2020. The long-run effects of monetary policy. NBER working paper no. 26666 .

Justiniano, A., Primiceri, G.E., Tambalotti, A., 2013. Is there a trade-off between inflation and output stabilization? American Economic Journal: Macroeconomics 5, 1–31.

King, R.G., Rebelo, S.T., 1993. Low frequency filtering and real business cycles. Journal of Economic Dynamics and Control 17, 207–231.

Kortum, S., 1993. Equilibrium r&d and the patent–r&d ratio: Us evidence. American Economic Review 83, 450–457.

Krishnamurthy, A., Vissing-Jorgensen, A., 2012. The aggregate demand for treasury debt. Journal of Political Economy 120, 233–267.

Mehrotra, N.R., 2018. Fiscal policy stabilization: Purchases or transfers? International Journal of Central Banking 14, 1–50.

Mertens, K., Ravn, M.O., 2013. The dynamic effects of personal and corporate income tax changes in the united states. American Economic Review 103, 1212–1247.

Moran, P., Queraltó, A., 2018. Innovation, productivity, and monetary policy. Journal of Monetary Economics 93, 24–41.

Nakamura, E., Steinsson, J., 2008. Five facts about prices: A reevaluation of menu cost models. The Quarterly Journal of Economics 123, 1415–1464.

Neiss, K.S., Nelson, E., 2003. The real-interest-rate gap as an inflation indicator. Macroeconomic Dynamics 7, 239–262.

Nuño, G., 2011. Optimal research and development and the cost of business cycles. Journal of Economic Growth 16, 257–283.

Palma, N., 2019. The existence and persistence of monetary non-neutrality: Evidence from a large-scale historical natural experiment. Technical Report.

Ramey, V.A., Zubairy, S., 2018. Government spending multipliers in good times and in bad: evidence from us historical data. Journal of Political Economy 126, 850–901.

Romer, C.D., Romer, D.H., 2004. A new measure of monetary shocks: Derivation and implications. The American Economic Review .

Smets, F., Wouters, R., 2007. Shocks and frictions in us business cycles: A bayesian dsge approach. American Economic Review 97, 586–606.

Stock, J.H., 2008. What's new in econometrics: Time series, lecture 7. Short course lectures, NBER Summer Institute .

Stock, J.H., Watson, M.W., 2012. Disentangling the channels of the 2007–09 recession. Brookings Papers on Economic Activity .

Stock, J.H., Watson, M.W., 2017. Identification and estimation of dynamic causal effects in macroeconomics. Working Paper .

Taylor, J.B., 1993. Discretion versus policy rules in practice, in: Carnegie-Rochester conference series on public policy. volume 39, pp. 195–214.

Wieland, J.F., Yang, M.J., 2016. Financial dampening. Technical Report. National Bureau of Economic Research.

Woodford, M., 2003. Interest and prices: Foundations of a theory of monetary policy. Princeton University Press Princeton, NJ.